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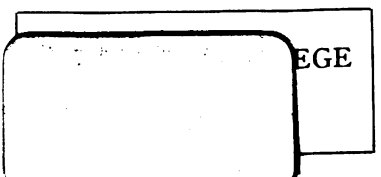

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CHILDREN'S ARITHMETICS BY GRADES

GLOBE SERIES

FOURTH BOOK

FIFTH YEAR

ESSENTIAL ELEMENTS

BY

WILLIAM E. CHANCELLOR, M.A.

SUPERINTENDENT OF SCHOOLS, BLOOMFIELD, N.J.

GLOBE SCHOOL BOOK COMPANY

NEW YORK AND CHICAGO

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"At about eight or nine there begins a new period, which for nearly four years constitutes a unique stage of life. . . . Child study suggests very plainly that this period should be devoted mainly to drill, habituation and mechanism. . . . Discipline should be the watch-word here."

President G. STANLEY HALL, LL.D.,
Clark University.

From Address on "The Ideal School,"
Council, National Educational Association, 1901.

MANHATTAN PRESS
474 WEST BROADWAY
NEW YORK

PREFACE

This book is for boys and girls who know thoroughly addition, subtraction, multiplication, and division, who understand clearly what common fractions and decimals are, who are familiar with counting and comparison, and who can use in simple problems the facts of weights and measures. For these the new work here includes the multiplication and division of fractions by fractions and of decimals by decimals, and further extension and application of the principles of ratio, proportion, percentage, cancellation, and equation. The rest of the book consists of drills and tests in the use of the fundamental and intermediate principles involved in the theory of practical arithmetic.

The plan of this book is to treat a topic and then to review it in connection with problems in preceding topics. One topic after another is taken up with such thoroughness as seems appropriate to the stage of the pupil's advancement; and some of the topics are repeated once or twice in the course of these pages. This plan secures an essentially spiral treatment. Subjects are introduced, but by no means exhausted before we pass to new subjects or to old subjects displayed in new matter.

It is sound child psychology to believe that boys and girls in fourth, fifth, and sixth grades have singular power and even pleasure in undergoing the exercises of drill in any and every subject. It is fortunate that there is such a period in the mental and physical life as that passed by children usually in their fourth, fifth, and sixth grades at school. It is unfortunate that sufficient advantage has not yet been taken of this age by the text-book writers. In particular in arithmetic we have many excellent books and monographs discussing

methods, devices, and exercises in teaching the elementary facts of number from one to twenty, and very many books discussing and exemplifying arithmetic as a science. Most of these latter books are logical, but not psychological. They are treatises upon arithmetic, manuals for adult teachers rather than for young students. But in a bibliography of many hundred titles one finds not ten books and essays which even try to present or to demonstrate proper methods of presenting to boys and girls nine to twelve years of age such fundamental and intermediate operations as are indicated in this book. Our pedagogical philosophers have clear and correct ideas of the minds of six year old children; and our text-book writers have at least scientifically arranged the topics of arithmetic for fifteen year old boys and girls. But I fail to find any considerable number of authors who have deliberately set before themselves the state of mind of the boys and girls who are likely to need such a book as this.

On the other hand, we find year by year our business men saying that the boys and girls who leave school at the ages of twelve to fourteen know nothing accurately in arithmetic, not even the fundamental operations. This is a curious fact in view of the other fact that the very power of the ten to twelve year old child is to undergo drill. This book is an effort to present interesting forms of such drill as will develop quickness and accuracy.

It is not meant that we are to question the principle that "the problem is the unit of arithmetic." It is meant that success in the problem is conditioned by mere mechanical facility and correctness in number-manipulation. There ought to be many problems in considerable variety in every arithmetic text-book. Every characteristic problem ought to be thoroughly studied and completely understood, and this as early in the child's life as possible.

Author and publishers desire to acknowledge the valuable suggestions of Superintendent Frank E. Spaulding, Ph.D., of Passaic, N. J., in reviewing the proofs of these pages.

SUGGESTIONS TO TEACHERS

1. The preface explains the general purpose of the book.

2. Read the book itself. The purposes of certain special features appear only when considered in relation to other features.

3. Read also Book III of this Series, which shows what pupils of this grade may be expected to know, or at least to have studied. Whenever the pupils of a class seem weak in matters necessarily preliminary to the grade in which they are, it is essential to review these matters both in instruction and in the pupils' own work. But never expect a class as a whole to know accurately and thoroughly every topic in any subject. Aim for perfect knowledge and proficiency, and one secures surprisingly good results from some pupils. Time may be wasted in undue drill of the whole class or in undue attention to individuals. It is very easy in arithmetic to spend an unnecessary amount upon single topics. A book upon the spiral plan, but which does not neglect thorough study of each topic as it is introduced or renewed, affords at frequent intervals an opportunity to find out the facts as to the conditions of the class and of its individual pupils.

4. It is unwise to ask children always to solve examples by the method of the book or by that of the teacher. Often children see through problems in ways distinctly individual. If their solutions are logical, correct, clear, and brief, we ought not only to accept but even to welcome them.

5. One kind of exercises is more valuable than any other in arithmetic: setting by pupils of their own problems and finding the solutions. Exactly as we encourage in our language-courses writing by the pupils themselves of various kinds of compositions, so also ought we to encourage the

making of their own examples. Such pages as these for drill in various topics are merely illustrations of exercises which may be set by the pupils for themselves. Every week there should be a period in the program for invention of problems by the pupils. One caution is to be observed. It is best for the teacher to inspect the problems to see that they are capable of solution. This invention of problems makes the pupils independent of printed answers and stimulates them to try proofs for themselves.

6. It is undesirable to pay much attention to the words of arithmetical rules and definitions at this stage lest the forms conceal the principles. Reasoning is the soul of arithmetic.

7. Incidentally such facts of the business world as the children see it for themselves may well be used. What facts will depend upon the neighborhood of the school and of the children's homes. These facts may be used as the bases of their invention of problems. This book uses standard facts such as may be seen almost anywhere. Continued observation of local facts will result in the accumulation of great stores of knowledge for use from time to time in the instruction and drill. Among the facts that ought to be known by children and are valuable as material for making problems are such as these; viz.:

(*a*) Railroad fares are usually three cents per mile. Children pay half fare. What are the costs for familiar distances? to great centers of population? for "excursion" and "return" tickets?

(*b*) Street-car fares are usually five cents. Children pay three cents. How many miles may one ride for one fare? Are several tickets sold at lower rates? Are special school children's tickets sold?

(*c*) Make market lists of prices for provisions, meats, groceries, shoes, dry goods, clothes, carpets, furniture, etc. Place these lists on the blackboard, and use them both in written and in oral work.

(*d*) Take imaginary trips, singly or in parties, paying all

expenses, — fares, food, purchases, admissions to places of amusement or recreation, etc.

(*e*) As far as possible, get the material from the children, but have it accurate.

8. It is unquestionable that a great variety of problems and exercises tends to make the principles themselves very clear. But avoid too much consecutive repetition of processes. We can secure the memory of processes best by variety of associations. Hence, though we ought to make each topic clear at its first presentation, we ought not to dwell too long upon it, but to go forward to the next, returning occasionally to refresh and to enrich our former knowledge.

9. Neatness in writing tends to accuracy in all exercises. Let us encourage excellent work with pen, pencil, or chalk crayon by commending it. Poor work may be due to defective eyesight, which causes increasing trouble with every added year of life. The figures written by most school children are too small and cramped.

10. In working through such a book as this it is well for a teacher to remember that in the early part of a term or year some problems may prove too hard for the pupils which they will be able to solve easily later in the term or in the year. If the problems are too easy, it will be found a very simple matter to increase the difficulties by adding another step or by dealing with larger figures. While the class is studying the topics, the teacher is studying the class; and while the class improves in industry and ability and technical knowledge, the teacher improves in instruction and in assignment of work through greater familiarity with the needs and powers of the individuals in the class.

11. In the practical use of this book a greater or a less proportion of the problems will be found too difficult for even the best pupils, not to say the average pupils, upon the first consideration of them.

(*a*) Sometimes it will be found advisable to give oral instruction regarding a problem.

(*b*) Many of the class will be able to return a few weeks later to a hitherto unsolved problem with power to solve it unaided by the teacher.

(*c*) Where pupils fail upon many problems, there may be need of a systematic review, for the purpose of developing in the imagination the concrete bases of arithmetic which are given by ratio and counting. But more often it is the better plan to go forward, using oral problems that involve the processes suitable to the ages and grade of the pupils. By mere change of numbers almost any problem may be made oral. Many classes are weakened in activity, power, and memory by too much oral instruction.

12. A book in which every problem is easy for nearly every child is too easy to give proper inducement for earnest, steady, upbuilding effort on the part of its students. The plan here has been to make a systematic outline, leaving to the teacher's judgment of the daily needs of the class exactly the number of problems and upon exactly what pages they should be for the ever insistent "next lesson." Something new for curiosity and a good deal that is old for power and for skill make the right combination usually for lessons in any subject. From a half page to a page will be usually a reasonable lesson, including both class and home work.

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TABLES

Counting Measure

12 units = 1 dozen	12 = 1 doz.
12 dozen = 1 gross	12 doz. = 1 gr.
12 gross = 1 great gross	12 gr. = 1 G. gr.
20 units = 1 score	20 = 1 sc.

Square Measure

144 square inches = 1 square foot	144 sq. in. = 1 sq. ft.
9 square feet = 1 square yard	9 sq. ft. = 1 sq. yd.
$30\frac{1}{4}$ square yards = 1 square rod	$30\frac{1}{4}$ sq. yd. = 1 sq. rd.
160 square rods = 1 acre	160 sq. rd. = 1 A.
43,560 square feet = 1 acre	43,560 sq. ft. = 1 A.
640 acres = 1 square mile or section	

Cubic Measure

1728 cubic inches = 1 cubic foot	1728 cu. in. = 1 cu. ft.
27 cubic feet = 1 cubic yard	27 cu. ft. = 1 cu. yd.

Cord Measure

16 cubic feet = 1 cord foot	16 cu. ft. = 1 cd. ft.
128 cubic feet = 1 cord	128 cu. ft. = 1 cd.

Time Measure

60 seconds = 1 minute	60 sec. = 1 min.
60 minutes = 1 hour	60 min. = 1 hr.
24 hours = 1 day	24 hr. = 1 da.
7 days = 1 week	7 da. = 1 wk.
12 months = 1 year	12 mo. = 1 yr.
30 days count as 1 month usually	30 da. = 1 mo.
365 days count as 1 year usually	365 da. = 1 yr.

TABLES

Liquid Measure

4 gills	= 1 pint	4 gi. = 1 pt.
2 pints	= 1 quart	2 pt. = 1 qt.
4 quarts	= 1 gallon	4 qt. = 1 gal.

Dry Measure

2 pints	= 1 quart	2 pt. = 1 qt.
8 quarts	= 1 peck	8 qt. = 1 pk.
4 pecks	= 1 bushel	4 pk. = 1 bu.

Linear Measure

12 inches	= 1 foot	12 in. = 1 ft.
3 feet	= 1 yard	3 ft. = 1 yd.
$5\frac{1}{2}$ yards	= 1 rod	$5\frac{1}{2}$ yd. = 1 rd.
$16\frac{1}{2}$ feet	= 1 rod	$16\frac{1}{2}$ ft. = 1 rd.
320 rods	= 1 mile	320 rd. = 1 mi.
5280 feet	= 1 mile	5280 ft. = 1 mi.

Avoirdupois Weight

16 ounces	= 1 pound	16 oz. = 1 lb.
100 pounds	= 1 hundredweight	100 lb. = 1 cwt.
2000 pounds	= 1 ton	2000 lb. = 1 T.

2240 pounds = 1 long ton (used for weighing coal at the mines and for merchandise at United States Custom Houses).

Customary Weights

1 bushel of potatoes	= 60 pounds
1 bushel of wheat	= 60 pounds
1 barrel of flour	= 196 pounds

Circular Measure

60 seconds	= 1 minute	$60'' = 1'$
60 minutes	= 1 degree	$60' = 1°$
90 degrees	= 1 quadrant	$90° = 1$ quad.
360 degrees	= 1 circumference	$360° = 1$ circ.

FACTS FOR PROBLEMS

A **brick** is 8 in. × 4 in. × 2 in. A **flatiron** weighs 6 lb.

1 M. **shingles**, laid 4 in. to the weather, cover 100 sq. ft.

A **lath** is 4 ft. × 1½ in. A **clapboard** is 4 ft. × 6 in.

A **roll of wall paper** is 24 ft. × 18 in.

A **cubic foot of water** weighs 1000 oz.; of **ice**, 930 oz.

Passenger trains move at the rate of 40 mi. per hour.

Fast mail trains can make a mile a minute.

The **greatest speed of the trotting horse** is a mile in two minutes.

At sea level the **weight of air** upon 1 sq. in. of surface is 14.7 lb.

The **cost of life insurance** in "old-line" companies per $1000 is: At 26 yr. of age, $21; at 38 yr., $30; at 50 yr., $48.50.

Letters are 2¢ per oz. and each fraction of ounce.

Newspapers and **periodicals**, when mailed by publishers, cost 1¢ per lb. In the ordinary course of the mail they cost 1¢ per 4 oz.

Printed matter, unsealed, costs 1¢ per oz.

Merchandise costs 1¢ per oz.

Telegrams cost 25¢ for 10 words, with 2¢ for each additional word, for distances within the local telegraph district; within the next "zone," 40¢ for 10 words; etc.

A dog weighs 40 lb.; a man, 150 lb.; a horse, 1000 lb.; an elephant, 8000 lb.; a locomotive, 150,000 lb.; an ocean steamship, 20,000,000 lb.

Sound is a radiating energy, causing waves that travel 1080 ft. per sec. in the air and 4700 ft. per sec. in the ocean.

Light is also a radiating energy, whose waves travel 196,000 mi. per sec. through the ether.

Sun time varies 1 hr. in 800 miles, latitude of New York City.

INTRODUCTORY REVIEW RECITE

1. One man's fortune was $\frac{10}{7}$ that of another man. The second man was worth, in the money value of his property, $700,000. What was the first man worth?

2. A farmer sold 8 cords of wood to one buyer, $2\frac{1}{2}$ times as much to another buyer, and $\frac{3}{4}$ as much to a third. How much wood did he sell to each of the second and third buyers?

3. What is an integer? a fraction? Give examples of integers, and of various kinds of fractions.

4. Why cannot we at once add unlike fractions?

5. A boy who had $\$2\frac{1}{2}$ paid $\$1\frac{3}{4}$ for three fine hens. How many dollars had he left? Was this more or less than one dollar? By how much?

6. What is meant by the expression, "Reducing fractions to lowest terms"? Explain the process.

7. What fractions are equivalent to $\frac{1}{2}$? What to $\frac{1}{3}$?

8. What is ratio?

9. What do we do when we measure things?

10. How many cord feet are there in 10 cords?

11. What line is the hypotenuse of a right angle triangle?

12. Give the tables of Long Measure, Square Measure, and Cubic Measure.

13. On an Italian coin are these letters, MDCCCLIV. In what year was it minted?

ADDITIONS

	A	B	C	D	E	F	G	H	I
J	1	2	3	4	5	6	7	8	9
K	2	3	4	5	6	7	8	9	1
L	1	9	3	4	7	6	5	8	2
M	3	4	5	6	1	8	9	7	2
N	1	2	3	4	5	9	9	6	8
O	4	5	6	7	8	9	9	5	1
P	2	1	3	4	5	8	8	5	7
Q	5	6	7	8	9	7	8	5	3
R	1	2	3	4	5	6	7	4	8
S	6	7	8	9	5	6	8	5	3
T	1	3	2	4	4	5	8	5	6
U	7	8	9	4	3	4	8	4	2
V	1	2	3	3	2	3	7	4	5
W	8	9	2	4	1	3	7	4	5
X	1	2	2	5	2	3	7	3	4
Y	9	1	3	6	4	1	6	2	8
Z	1	1	3	7	4	6	5	1	2

Prove by adding from both top and bottom or from both left and right.

1. Add by columns. 2. Add by rows.

3. Add diagonally: from AJ to IR; from IR to AZ.

4. Add from left to right and from right to left.

5. Note easy combinations. $5+5=10$, $4+4+2=10$, $7+7+7=21$, $8+7=15$, $4+1+3+7=15$, etc.

6. Add orally in recitation; and in writing.

7. Add two, three, or more columns written together:

12	23	34	567	6789
23,	34,	45,	678,	7891, etc.

MULTIPLICATIONS

	A	B	C	D
E	$2 \times 2 = ?$	$3 \times 2 = ?$	$2 \times 4 = ?$	$3 \times 6 = ?$
F	$4 \times 3 = ?$	$5 \times 3 = ?$	$4 \times 5 = ?$	$5 \times 7 = ?$
G	$6 \times 4 = ?$	$7 \times 4 = ?$	$6 \times 6 = ?$	$7 \times 9 = ?$
H	$8 \times 5 = ?$	$9 \times 5 = ?$	$8 \times 7 = ?$	$9 \times 8 = ?$
I	$10 \times 6 = ?$	$11 \times 6 = ?$	$10 \times 8 = ?$	$11 \times 7 = ?$
J	$12 \times 7 = ?$	$13 \times 7 = ?$	$12 \times 9 = ?$	$13 \times 6 = ?$
K	$14 \times 7 = ?$	$15 \times 7 = ?$	$14 \times 9 = ?$	$15 \times 5 = ?$
L	$16 \times 6 = ?$	$17 \times 6 = ?$	$17 \times 8 = ?$	$17 \times 4 = ?$
M	$18 \times 5 = ?$	$19 \times 5 = ?$	$18 \times 7 = ?$	$19 \times 3 = ?$
N	$20 \times 4 = ?$	$21 \times 4 = ?$	$20 \times 6 = ?$	$21 \times 2 = ?$
O	$22 \times 3 = ?$	$23 \times 3 = ?$	$22 \times 5 = ?$	$23 \times 3 = ?$
P	$24 \times 2 = ?$	$25 \times 2 = ?$	$24 \times 4 = ?$	$25 \times 4 = ?$
Q	$12\frac{1}{2} \times 8 = ?$	$16\frac{2}{3} \times 6 = ?$	$33\frac{1}{3} \times 3 = ?$	$37\frac{1}{2} \times 2 = ?$
R	$48 \times 2 = ?$	$20 \times 5 = ?$	$8\frac{1}{3} \times 12 = ?$	$66\frac{2}{3} \times 3 = ?$
S	$12\frac{1}{2} \times 2 = ?$	$16\frac{2}{3} \times 3 = ?$	$12 \times 8 = ?$	$100 \times \frac{1}{2} = ?$
T	$3 \times 3 = ?$	$4 \times 4 = ?$	$5 \times 5 = ?$	$7 \times 7 = ?$
U	$8 \times 8 = ?$	$9 \times 9 = ?$	$12 \times 12 = ?$	$11 \times 11 = ?$
V	$15 \times 15 = ?$	$18 \times 18 = ?$	$20 \times 20 = ?$	$24 \times 24 = ?$
W	$25 \times 25 = ?$	$50 \times 50 = ?$	$100 \times 100 = ?$	$250 \times 4 = ?$

EASY METHODS

1. $98¢ \times 9 = ?$ $100¢ - 98¢ = 2¢$ $100¢ \times 9 = 900¢$ $2¢ \times 9 = 18¢$ $900¢ - 18¢ = 882¢ = \$8.82$.

2. $25¢ \times 35 = ?$ $25¢ \times 4 = 100$ $100¢ \times 35 = 3500¢$ $3500¢ \div 4 = 875¢ = \$8.75$.

3. $49¢ \times 10 = ?$ $49¢ = \$.49$ $\$.49 \times 10 = \$4.9 = \$4.90$.

4. $4\frac{1}{2}¢ \times 14 = ?$ $4\frac{1}{2}¢ = 9¢ \div 2$ $9¢ \times 14 = 126¢$ $126¢ \div 2 = 63¢$ or $4\frac{1}{2}¢ \times 2 = 9¢$ $14 \div 2 = 7$ $4\frac{1}{2}¢ \times 14 = 9¢ \times 7 = 63¢$.

QUESTIONS

1. What will 8 yards of sheeting cost at $12\frac{1}{2}¢$ a yard?
2. What will 9 pounds of prunes cost at $9¢$ a pound?
3. What is the cost of 7 lb. of butter at $22¢$ per lb.?
4. What is the cost of 12 pounds of tea at $60¢$ a pound?
5. What is the cost of 12 lb. of coffee at $30¢$ per lb.?
6. What is the cost of 11 yards of cloth at $4 a yard?
7. What is the cost of 9 books at $40¢$ each?
8. At $12 a cord, what will 12 cords of wood cost? 10 cords? 9 cords? 7 cords? 6 cords? 5 cords?
9. How many feet are there in 11 yards? 10 yards? 29 yards? 48 yards? 97 yards?
10. How many quarts are there in 10 gallons? 9 gallons? 8 gallons? 50 gallons?
11. If you buy 5 marbles for $1¢$, how many marbles can you buy for $11¢$? $8¢$? $1.00?
12. At $8¢$ a quart, what will 12 quarts of milk cost? 11 quarts? 6 quarts? 24 quarts? 98 quarts? 17 quarts?
13. What will be the cost of 12 yards of braid at $10¢$ a yard? 10 yards? 60 yards? 8 yards? 25 yards?
14. What is the cost of 8 yards of silk at $5 a yard?
15. If 7 yards of cloth make a suit, how many yards will it take to make 8 suits?
16. How much do 9 pounds of ginger cost at $30¢$ a pound?
17. At $12 apiece, what will 10 sofas cost?
18. What is the cost of 10 bbl. of vinegar at $9 per bbl.?
19. What will 11 pair of shoes cost at $5 a pair?
20. If 8 men can build a wall in 9 days, how long will it take 1 man to build it?

NOTATION

Notation is a method of writing numbers.

The system of notation in general use is called the Arabic Notation. It was introduced into Europe in the twelfth century by the Arabs who founded the empire of the Moors in Spain. It came to Arabia from India.

This notation uses ten characters of figures to express numbers. It is also called a decimal notation, because the numbers increase and decrease in a tenfold ratio.

Figures,	1	2	3	4	5	6	7	8	9	0
Names,	one	two	three	four	five	six	seven	eight	nine	cipher

The last figure, or character, is also called zero, or naught. It has no value when standing alone. The first nine figures are termed significant figures, as each always signifies a value of its own. These figures are also called digits.

We express ten units by using not one figure, but two figures, 10. The cipher occupying the first place, at the right hand, gives a tenfold value to the 1, and shows that there are no units of the first order, or place. The 1 ten is a unit of the second order.

The number 100, equal to 10 tens, is a unit of the third order, and, therefore, the digit 1 occupies the third place. In a decimal system of notation ten units of any order make one unit of the next higher order.

1 unit of the first order is expressed by . . . . 1.
1 unit of the second order is expressed by . . . . 10.
1 unit of the third order is expressed by . . . . 100.
1 unit of the fourth order is expressed by . . . . 1000.
1 unit of the fifth order is expressed by . . . . 10000.
1 unit of the sixth order is expressed by . . . . 100000.

Units of higher orders are written in the same way.

How many units of what order are in 70000000?

NUMERATION

Numeration is a method of reading numbers. In order that numbers may be read conveniently, they are grouped into periods of three orders each.

Learn the names of the periods in the following table:

Hundreds of Quintillions. Tens of Quintillions. Quintillions.	Hundreds of Quadrillions. Tens of Quadrillions. Quadrillions.	Hundreds of Trillions. Tens of Trillions. Trillions.	Hundreds of Billions. Tens of Billions. Billions.	Hundreds of Millions. Tens of Millions. Millions.	Hundreds of Thousands. Tens of Thousands. Thousands.	Hundreds. Tens. Units.
3 6 3,	1 5 4,	7 9 2,	8 8 3,	2 5 1,	3 6 6,	7 2 4.
Period of Quintillions.	Period of Quadrillions.	Period of Trillions.	Period of Billions.	Period of Millions.	Period of Thousands.	Period of Units.

The periods in their order above Quintillions are Sextillions, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, Tredecillions, Quattuordecillions, etc., to Vigintillions. Numbers higher than these are seldom needed, except by astronomers in measuring the distances of the stars.

If a number consists of more than one period, the left-hand period may contain one, two, or three figures, but all the other periods must have three figures in each period. Vacant orders are filled by ciphers.

The system of numeration in use in England has six orders in every period. Hence in the English system a million is a thousand thousand, a billion is a million times a million, a trillion is a million times a billion, and so on.

Point off the number into periods of three figures each, beginning at the right hand, and read each period as if it stood alone, naming its order.

Do not use the word *and* in reading whole numbers.

Ciphers are read only as they show the values of higher figures.

1. Read:

7849630	10000001	309025607	7070000423
341328	20220022	980076011	91234013402
418254	92568987	800309560	91234267109
827171	17643000	987654321	670000000069
6850406	80970503	967004296	1001001001001
5931081	91131140	9738413208	61300400007623
7546738	30180070	5707068080	81008100810081
8080808	800309560	2560530200	100000200993004
48726870634103264		23200300500	1001987000100000

2. The world has 1,500,000,000 people, and $1,000,000,000,000 worth of wealth. Write this statement in words.

3. Write in figures the following numbers:

Twenty-eight thousand, three hundred nineteen.

Three million, one hundred forty-five thousand, six hundred eighty-five.

Eighty-six million, four hundred three thousand seven.

Nine hundred nine million, ninety-nine thousand nine.

One hundred forty million, six hundred two thousand seven.

Seven hundred million, twenty thousand.

Five billion, five million, five thousand five.

Sixty trillion, sixty million seventy.

Seventeen trillion, seven million seventy.

Eight billion, seventy million, four hundred thousand, two hundred seventy-six.

DECIMAL FRACTIONS

The **powers** of a number are its products when multiplied one or more times by itself. The *first power* is the number itself.

$2 \times 2 = 4$. 4 is the second power of two

The **powers** of 10 are 10, 100, 1000, 10,000, etc. These are called the *first*, *second*, *third*, *fourth* . . . *powers* of ten.

$10 \times 10 \times 10 = 1000$. 1000 is the third power of 10.

A fraction which has for its denominator one of the powers of 10 is called a **decimal fraction**, or a **decimal**. All other fractions are called *common fractions*.

To simplify the writing of the denominators of decimal fractions, a method of notation is used by which we express the value of the denominator in every case by the decimal form of writing the numerator.

.3 stands for $\frac{3}{10}$, and is read three tenths.

.25 stands for $\frac{25}{100}$, and is read twenty-five hundredths.

.347 stands for $\frac{347}{1000}$, and is read three hundred forty-seven thousandths.

In each case the figures that follow the *decimal point* form the numerator of the fraction.

The number of figures that follow the *point* corresponds to the number denoting the particular power of 10 that forms the denominator of the fraction.

As the first power of 10 is 1 followed by zero, and the second power of 10 is 1 followed by two zeros, and the third power of 10 is 1 followed by three zeros, etc., in every case we can write the denominator by affixing to 1 a number of zeros equal to the number of figures that follow the decimal point.

.426789 stands for $\frac{426789}{1000000}$.

Six zeros are affixed to the 1, because the number of figures that follow the decimal point is in this case six. This number is read four hundred twenty-six thousand seven hundred eighty-nine millionths.

$\frac{275}{1000} = .275$ $\frac{36}{1000} = .036$ $\frac{743}{100000} = .00743$

.00025 stands for $\frac{25}{100000}$ .025707 stands for $\frac{25707}{1000000}$

The ciphers that come between the point and the figures 25 are not written in the numerators of the common fractions, for 00025 equals 25, and 025707 equals 25707.

The decimal point changes the value of figures.

Any common fraction, having as a denominator a power of 10, can be written as a decimal, by writing the numerator only, and so placing the decimal point that it shall have as many figures on the right hand as there are ciphers in the denominator of the given fraction. If the numerator does not contain as many figures as there are ciphers in the denominator, the lack must be supplied by placing as many ciphers between the point and the given numerator as are needed.

1. Write as decimals:

a. $\frac{3}{10}$. *d.* $\frac{9}{10000}$. *g.* $\frac{27}{100}$. *j.* $\frac{3287}{100}$. *m.* $\frac{319}{1000}$.
b. $\frac{5}{100}$. *e.* $\frac{38}{1000}$. *h.* $\frac{47}{1000000}$. *k.* $\frac{4631}{1000}$. *n.* $\frac{4374}{10000}$.
c. $\frac{7}{1000}$. *f.* $\frac{502}{1000}$. *i.* $\frac{86532}{100}$. *l.* $\frac{976}{100000}$. *o.* $\frac{397}{10}$.

2. Change to common fractions:

p. .079 *r.* .007 *t.* .00003 *v.* .15698 *x.* .8 *z.* .193
q. .0321 *s.* .5146 *u.* .0078 *w.* .000301 *y.* .0099 *&.* .00307

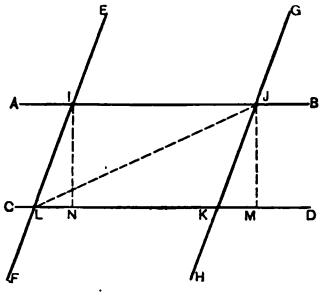
3. Decimal means tenth. Why is it a suitable word for this kind of fractions?

4. How many times do we multiply ten by itself to produce 10,000? 1,000,000? 1,000,000,000?

FIGURES

Lines that can never meet however far they may be extended are **parallel.**

When two pairs of parallel lines cross each other, the figure made by them is called a **parallelogram.**



AB and *CD* are *parallel.*
EF and *GH* are *parallel.*
The pairs of parallels cross.
What is the figure *IJKL*?

The line connecting the opposite angles of a parallelogram is a **diagonal.** What is *LJ*?

A perpendicular line between the parallels of a parallelogram gives its **altitude.** What is the *altitude* of *IJKL*?

The *rectangle IJMN* is equal in *area* to the *parallelogram IJKL.*

Draw these figures on paper. Cut out the triangle *JMK.* Does it equal the triangle *ILN*? If the figure *IJKN* is a part of both *IJMN* and *IJKL*, what does the equality of the triangles show?

The *area* of any *rectangle* equals the product of the numbers that measure its *length* and *breadth*.

The *area* of any *parallelogram* equals the product of the numbers that measure its *base* and *altitude*.

The **base** of a parallelogram is either of the parallel sides between which the *altitude* is taken.

LK or *IJ* is the *base* of *IJKL.*

Draw these parallelograms, and find their altitudes, bases, and areas:

1. 6 in. × 2 in.	2. 5 in. × 3 in.	3. 8 in. × 4 in.
4. 9 in. × 4 in.	5. 10 in. × 2 in.	6. 12 in. × 5 in.
7. 8 in. × 6 in.	8. 12 in. × 4 in.	9. 15 in. × 8 in.

Do not find the altitudes by measurements with fractions smaller than $\frac{1}{8}$ in. Draw some rectangles among these parallelograms.

THE DECIMAL NOTATION : TEACH

A number containing decimal units is called a decimal number or a decimal. The part at the right of the decimal point is called the **decimal** part; the part at the left is termed a **whole number** or **integer.**

In the number 6666, the four figures, beginning at the left and counting toward the right, have respectively these values: 6000, 600, 60, 6; the first 6 at the left having 10 times the value of the second; the second 6, 10 times the value of the third 6; etc. The values of the figures, counting toward the right, decrease in a tenfold ratio. But, beginning at units' place and counting toward the left, each figure has only one tenth of the value of the figure to the left of it. Any figure moved one place to the left is multiplied by 10, and any figure moved one place to the right is divided by 10.

When this notation is continued to the right of the decimal point, beyond units' place, as in 6666.6666 the value of the first figure to the right of units' is $\frac{1}{10}$ of 6, or $\frac{6}{10}$; the value of the next figure is $\frac{1}{10}$ of $\frac{6}{10}$, or $\frac{6}{100}$; the value of the next figure is $\frac{6}{1000}$; of the next figure $\frac{6}{10000}$.

The figures on the right of the decimal point may be written as common fractions, $\frac{6}{10}$, $\frac{6}{100}$, $\frac{6}{1000}$, $\frac{6}{10000}$, each of these having as a denominator some power of 10.

$\frac{6}{10} = .6$ $\frac{6}{100} = .06$ $\frac{6}{1000} = .006$ $\frac{6}{10000} = .0006$

DECIMAL FRACTIONS

A common fraction is a fraction whose denominator may be any number: as, 5, 9, 22, 859.

$\frac{3}{5}$, $\frac{5}{9}$, $\frac{17}{22}$, $\frac{37}{859}$, are common fractions.

A common fraction arises from dividing a unit into any number of equal parts. Its denominator must always be written.

A decimal fraction arises from dividing a unit or smaller number by 10, or some power of 10. Its denominator is not usually expressed in figures. $\frac{7}{10}$, $\frac{31}{100}$, $\frac{923}{10000}$, and .7, .31, .0923, are decimal fractions. The second is the more convenient style.

.0923 means 923 divided by 10000.

NUMERATION TABLE

etc., etc.	7th Order or Place,	6th Order or Place,	5th Order or Place,	4th Order or Place,	3d Order or Place,	2d Order or Place,	1st Order or Place,		1st Order or Place,	2d Order or Place,	3d Order or Place,	4th Order or Place,	5th Order or Place,	6th Order or Place,	7th Order or Place,	8th Order or Place,	9th Order or Place,	etc., etc.
	2	9	4	5	1	6	7	.	5	8	3	2	8	6	4	1	7	
	Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.	Decimal Point.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.	Hundred-millionths.	Billionths.	
	Whole Numbers.								*Decimal Fractions.*									

The number in the table is read,

Two million nine hundred forty-five thousand one hundred sixty-seven, and five hundred eighty-three million two hundred eighty-six thousand four hundred seventeen billionths.

Read the following:

a. 398.7561	*d.* 278934.1	*g.* 4326915.3	*j.* .000927
b. 38.42936	*e.* 12345681.98	*h.* 986.432	*k.* 293.42865
c. 1.532943	*f.* 29.87521985	*i.* .24387	*l.* .24139876

RECITE

VALUES OF DECIMALS

a. $.0675 = \frac{675}{10000} = \frac{600}{10000} + \frac{70}{10000} + \frac{5}{10000}$

$= \frac{6}{100} + \frac{7}{1000} + \frac{5}{10000}$

$= .06 + .007 + .0005$

$= .0675$

$= \frac{6}{\text{hundredths}} + \frac{7}{\text{thousandths}} + \frac{5}{\text{ten-thousandths}}$

Discuss by this method:

b. .349 *c.* .0028 *d.* .3486 *e.* .00387 *f.* .6036

Read each decimal as if it were a whole number, giving it the name of the order of its right-hand figure.

Read and discuss the following numbers:

g. .25	*n.* .341	*u.* .2374	*bb.* 327.264
h. .362	*o.* .0003	*v.* .2006	*cc.* 80.9056
i. .451	*p.* 45.7056	*w.* .0104	*dd.* .0000008
j. .5675	*q.* 12.07067	*x.* 42.05063	*ee.* 231.146
k. .8432	*r.* .6914	*y.* .00071423	*ff.* 4.3291
l. .003	*s.* .008196	*z.* 93.42	*gg.* 71.00089
m. .028	*t.* .0624	*aa.* 147.1934	*hh.* 167.193

DECIMALS

Write in figures:

1. Forty-one, and three tenths.
2. Sixteen, and three millionths.
3. Sixty-five, and fifteen thousandths.
4. Eighty, and three millionths.
5. Two, and three hundred millionths.
6. Four hundred, and ninety-two thousandths.
7. Three thousand, and twenty-one ten-thousandths.
8. Forty-seven, and twenty-one hundred-thousandths.
9. Fifteen hundred, and three millionths.
10. Thirty-nine, and six hundred forty thousandths.
11. Three thousand, and eight hundred forty millionths.
12. Six hundred, and fifteen thousandths.
13. Fifty-eight ten-thousandths.
14. One thousand, and nine hundred-thousandths.
15. Seven ten-thousandths.
16. 300 units, and three hundred-thousandths.
17. Seven hundred, and four millionths.
18. Nine thousand, and fifty-six millionths.
19. Sixty units, and 502784 millionths.
20. Six hundred thousand, and four millionths.
21. Twenty-seven, and four tenths.
22. Thirty-six, and fifteen thousandths.
23. Ninety-nine, and twenty-seven ten-thousandths.
24. Three hundred twenty thousandths.
25. Two hundred, and three hundred twenty millionths.
26. Three thousand six hundred ten ten-thousandths.

27. Read the following:

a. .32	*f.* 23.07	*k.* 41.603	*p.* 22.3628
b. .246	*g.* 12.683	*l.* 1.2864	*q.* 105.26
c. .3624	*h.* 20.064	*m.* 1000.115	*r.* 1000.3
d. 42.068	*i.* 35.0072	*n.* 871.29875	*s.* .2468953
e. 17.401	*j.* 67.4008	*o.* 2.3003	*t.* 10.000001

28. Write without denominators: WRITE

a. 462 thousandths.
b. 25 hundred-thousandths.
c. 1637246 ten-millionths.
d. 71 thousandths.
e. 261 hundred-thousandths.
f. 751 trillionths.
g. 2391 ten-thousandths.
h. 65 hundred-millionths.
i. 7 millionths.
j. 19 ten-thousandths.
k. 65 hundredths.
l. 121 millionths.

29. Write in figures:

a. Forty-nine hundred ten-thousandths.
b. Fifty-nine, and sixty-seven ten-thousandths.
c. Four hundred sixty-nine ten-thousandths.
d. Seventy-nine, and four hundred fifteen millionths.
e. One hundred five, and ninety-five ten-millionths.

OUR DECIMAL MONEY STUDY

In United States money the dollar is the unit, dimes are tenths of a dollar, cents hundredths, and mills thousandths; hence, any sum may be expressed in dollars and decimals of a dollar.

\$5 and 4 cents are \$5 and $\frac{4}{100}$; written also \$5.04.

\$20 and 96 cents and 8 mills are \$20, 96 hundredths and 8 thousandths of a dollar; written \$20.968.

The mill is not coined. It is used only in computation, or "figuring." 5 mills or more count as a cent.

ZEROS IN DECIMALS

$.3 = \frac{3}{10}$ $.03 = \frac{3}{100}$ $.003 = \frac{3}{1000}$

$.8 = \frac{8}{10}$ $.0008 = \frac{8}{10000}$ $8.070 = \frac{807}{100}$

But $.3 = .30 = .300 = .3000 = .30000,$

since $\frac{3}{10} = \frac{30}{100} = \frac{300}{1000} = \frac{3000}{10000} = \frac{30000}{100000}.$

and $003 = 03 = 3 = 3.0 = 3.00,$

since in each case we have three units and three only.

In a decimal fraction a zero between the *decimal point* and any digit of value at the right of the *point* shows the order of the digit and absence of value in the order filled by the zero.

Explain the zeros in:

I. .03507 II. .00581 III. .5700 IV. .901306

MULTIPLYING AND DIVIDING BY TEN

$.3 \times 10 = 3$ since $\frac{3}{10} \times 10 = 3$

a. $.5 \times 10 = ?$ *b*. $.8 \times 10 = ?$ *c*. $.7 \times 10 = ?$

$.65 \times 10 = 6.5$ since $\frac{65}{100} \times 10 = \frac{65}{10} = 6\frac{5}{10}$

d. $.89 \times 10 = ?$ *e*. $.54 \times 10 = ?$ *f*. $.93 \times 10 = ?$

g. $? \times 10 = 5.4.$ *h*. $? \times 10 = 8.7.$ *i*. $? \times 10 = 5.8.$

$.5 \div 10 = .05$ since $\frac{1}{10}$ of $\frac{5}{10} = \frac{5}{100}$

j. $.9 \div 10 = ?$ *k*. $.6 \div 10 = ?$ *l*. $3 \div 10 = ?$

In decimals notice always the DECIMAL POINT.

READING DECIMALS

By the rule of decimal notation a digit one place to the right of the point has ten times the value of the same digit two places to the right.

.5 or 5 tenths = 50 hundredths or 500 thousandths, and .596 may be read in any of the following ways:

$$.596 = \begin{cases} \text{5 tenths, 9 hundredths, and 6 thousandths;} \\ \text{or 59 hundredths and 6 thousandths;} \\ \text{or 5 tenths and 96 thousandths.} \\ \text{or 596 thousandths;} \end{cases}$$

MONEY AND DECIMALS

1. Express decimally:

a. $17 and 5 mills.
b. $215 and 8 cents.
c. $275 and 5 mills.
d. $9 and 8 mills.
e. $15 and 6¢ 9 m.
f. $27 and 18¢ 2 m.
g. $39 and 39¢ 7 m.
h. $37 and 26¢ 7 m.
i. 4 dollars and 6 mills.
j. 109 dollars and 1 mill.
k. 65 cents and 2 mills.
l. $12 and 3 mills.
m. $147 and 4 cents.
n. $148 and 4 mills.
o. $4 and 6 mills.
p. $9 and 6¢ 9 m.

2. How many mills are there in 67 cents? in $54?

3. How many cents are there in $125? in $1000?

4. In $400 are how many cents? how many mills?

5. In $375 are how many cents? how many mills?

6. How many mills are there in $4? in $6000? in $10?

7. In $71 45 cents 3 mills are how many mills in all?

8. Change:

a. 1607 mills to dollars, cents, and mills.
b. 170464 cents to dollars and cents.
c. 94780900 cents to dollars and cents.
d. 74164210 mills to dollars, cents, and mills.

9. Read the following:

.2	.306	.3551	2.02004
.02	.62854	.0986	121.300387
.002	.00016	.82008	2.3158218

REDUCTION OF DECIMALS

Annexing ciphers to a decimal does not alter its value.

1. Reduce 9 to tenths. 9.0 Read, *ninety tenths.*

2. Express 2.1 as thousandths. 2.100. Read, *two thousand one hundred thousandths.*

Two thousand *and* one hundred thousandths would mean 2000.100.

3. Write thirty-four hundredths as ten-thousandths. .3400. Read, *three thousand four hundred ten-thousandths.*

4. Reduce to ten-thousandths:

a. .87	*c.* .657	*e.* .06	*g.* .325	*i.* .334
b. .015	*d.* .17	*f.* .77	*h.* .087	*j.* .009

5. Reduce to millionths:

k. .7208	*m.* .9361	*o.* .0035
l. .7051	*n.* 1.327	*p.* 15.00789

6. Reduce to hundred-thousandths:

a. .246	*f.* 23.07	*k.* 67.4008	*p.* 100.024
b. .3624	*g.* 3.072	*l.* 42.068	*q.* 1.0024
c. 12.683	*h.* .124	*m.* 17.401	*r.* 6.0009
d. 20.064	*i.* .35	*n.* 47.0025	*s.* 7011.2
e. 2.009	*j.* 35.0072	*o.* 53.734	*t.* 80.0056

RECITE

7. What part is .09 of .9? .007 of .7? .08 of 8?
8. What is the ratio of .6 to 6? of 9 to 9?
9. What is the ratio of 1 to .01? of .3 to .03?
10. What is the purpose of the zero in .305? 4.50?
11. What per cent is \$.9 of \$9? \$.4 of \$.8?
12. What fraction corresponds with per cent?
13. What is 10% of 1 expressed decimally? of .01?
14. Read the following:

a. .1 qt. *b.* .3 cd. *c.* .08 mi. *d.* .5 pk. *e.* \$.01.

ADDITION OF DECIMALS

1. Add 2.0357, 120.15, .037, 19, and 70.004.

```
  2.0357
120.15
   .037
 19.
 70.004
--------
211.2267
```

In adding decimals set the figures for units of the same order in columns as in adding numbers without decimals.

2. Add 80.007, .0091, 49.375, 1001.25, 407.7125.

```
   80.007
     .0091
   49.375
 1001.25
  407.7125
-----------
1538.3536
```

Notice that the decimal points are in vertical line with each other.

3. Add by columns and by rows:

	I	II	III	IV
A	3.406	45.372	300.	4.7561
B	44.04	3650.	4.12305	.000375
C	671.75	.00003675	4120.	8.125
D	376.15	40000.	800.01725	.0125
E	43.702	30102.5	.46125	3.67003
F	4.01	.003	29.	1.2345
G	.012	32.1	4.12	.00036
H	.00023	.00967	.00476	.0391
I	39.102	.046	471.2	82.137
J	8170.007	3.295	014.	121.0258

4. Mr. Brown bought a horse for $100, a carriage for $172.50, a whip for $1.375, and a carriage robe for $3.375. What was the total amount of his purchases?

DECIMALS IN REVIEW

1. Find the sum of:

a. 43.006, .73, .0006, 4.

b. 7.89, 46.306, .000674, .001.

c. 17.00013, 360, .007, 4.09.

d. 19.0406, 173.00009, 3.5, 1000.

e. 92.183, .706302, 400, 1.7496, .0406739.

f. 100.01, 1.0001, 1000.0034, 10000, 88898.9865.

g. .0698, 30.74, 2076.940635, .00473296, 1.09, 400.06.

h. 13.97643, .00917, 4.96, 31.7049631, 4789, .3406.

i. 1400, 3.003, .79867, 512.6345, .000987, 99.98.

j. 55.55, .005555, .5555, .555, 5.00005.

2. Answer the following:

k. $478.63 + 89.005 + 5437.25 + .000125 + 541.2375 = ?$

l. $6.75 + 89.5 + .00009 + .502 + 7.548 + 2178 = ?$

m. $51.416 + 915.13 + .04751 + .005819 + .000045.$

n. $74.874 + 81.33486 + .0974 + 1.0718 + 88.71034 = ?$

o. $61.371568 + .0091 + .06371 + .0008714 + 43.814 = ?$

p. $75.67 + 125 + .00725 + 67 + 19.25 + 37 + 2145.0007 = ?$

3. Find the value of:

q. $91.75 + 543.175 - 89.005 - 17.007 + 161.1003 - 317.$

r. $17.48 - 1.586 + .374 + .00076 - 41.9 + 157.01.$

s. $.0078 - .172 - .78154 + 37.011 - 11.9901 + 169.999.$

t. $51.16 - .0073 + 89.4 - 3.90909 + 41.051 - 2.09091.$

u. Multiply .15 by 100 and divide the product by .025.

v. From 10 subtract a thousandth part of 1.

w. Subtract a thousandth part of 10 from its hundredth part.

x. If the sum of 1.015 and .001015 be added to their difference, how much will the total fall short of 1000?

y. Find the perimeter of a square whose side is 3.25 in.

SUBTRACTION OF DECIMALS

1. Subtract 9.7658 from 117.053.

$$\begin{array}{r} 117.053 \\ 9.7658 \\ \hline 107.2872 \end{array}$$

In subtracting decimals set the figures for units of the same order in columns.

2. Subtract 4.12575 from 21.

$$\begin{array}{r} 21. \\ 4.12575 \\ \hline 16.87425 \end{array}$$

The process is this:

$$\begin{array}{r} 10\ 9\ 9\ 9\ 9 \\ 1\not{0}\ \not{1}\not{0}\ \not{1}\not{0}\ \not{1}\not{0}\ \not{1}\not{0}\ 10 \\ \not{2}\not{1}.0\ 0\ 0\ 0\ 0 \\ 04.1\ 2\ 5\ 7\ 5 \\ \hline 16.8\ 7\ 4\ 2\ 5 \end{array}$$

3. Subtract:

a. 3.594 from 13.007.
b. .0976 from 1.28.
c. 13.265 from 100.0008.
d. 432.125 from 678.427.
e. 36.75 from 40.1.
f. 100.376 from 451.86975.
g. 3.07625 from 500.8.
h. .00061765 from .001.
i. 5.056 from 20.91.
j. .98976 from 1.
k. 18.7346908 from 90.32064.
l. 412.865 from 503.125.
m. 4.38675 from 5.
n. .000625 from 30.
o. 473.5 from 1000.
p. .00384752 from 012345.

4. Find the value of:

q. 71.48 − 35.71.
r. .00078 − 000089.
s. 87.31 − 6.871496.
t. 1.000009 − .784163.
u. 75 − .0057.
v. 4.001 − .374.

5. Mr. Smith had $875 in cash and paid as taxes on three pieces of property, $251.654, $401.736, and $105,108. What amount was left?

6. Mr. Williams owned .1 A. of land and sold from it .05 A. How much land had he then?

7. $.25 − $.12 + $.405 + $.555 − $1 = ?

REVIEW

1. A man sold .375 of his farm. How much of it had he left?

2. How many sevenths of a pint are there in 3 pints?

3. If 35 marbles be divided equally among 5 boys, how many marbles will 2 of the boys get?

4. A man who had $1000 in the bank withdrew $450.25. How much had he there then?

5. A grocer who had a hogshead of molasses lost 215 thousandths of it by leakage. How much was left?

6. When a circle is divided into 5 equal parts, what is one part called? What are 4 of the parts called?

MULTIPLICATION DRILL TABLE

In the table below give the product of each pair of consecutive digits, both in columns and in rows.

With such a row as 4, 2, 6, 9, 5, 0, 1, 8, 6, 4, 3, 5, say only "8, 12, 54, 45, 0, 0, 8, 48, 24, 12, 15."

	A	B	C	D	E	F	G	H	I	J	K	L
I	2	4	6	8	9	7	6	3	1	4	9	7
II	7	1	5	4	1	8	2	6	4	7	2	1
III	3	7	6	1	7	1	5	8	9	0	1	2
IV	1	8	3	4	5	6	2	7	8	9	3	4
V	9	4	1	1	8	1	9	3	4	5	6	7
VI	4	3	9	8	7	2	0	2	3	4	5	6
VII	5	6	2	8	8	2	1	8	9	0	2	3
VIII	6	2	1	8	9	2	2	5	6	7	8	9
IX	8	1	8	9	0	2	3	4	2	1	8	4
X	7	0	1	9	1	2	6	7	1	8	2	4
XI	1	5	5	2	9	3	6	7	8	9	0	3
XII	4	8	7	6	3	3	3	4	5	6	7	8

DRY MEASURE

The standard unit of *dry measure* is the bushel.

It is $18\frac{1}{2}$ inches in diameter and 8 inches deep, and contains 2150.42 cu. in.

The liquid quart contains $57\frac{3}{4}$ cu. in. and the dry quart $67\frac{1}{5}$ cu. in. What is the difference?

REVIEW

Add:

1.	2.	3.	4.
562347	347326	876208	6752
85476	587765	343672	784519
36728	768246	394625	22687
6544	87335	4713	456275
97328	654	29875	44824
475564	363578	368207	612785

5. How many days would a man take to walk 156 miles at the rate of 3 miles an hour for 8 hours a day?

6. Which would fill the larger space, 20 qt. of milk or 20 qt. of corn?

7. How far would a steamer go in 9 days, of 24 hours each, at the rate of 15 miles an hour?

8. $7.25+608.125+69.785+25600.5+.506+.00525=?$

9. $39.5+10.25+689.75+415.5+.00075+6085.5=?$

10. $161700\div15=?$ $585900\div27=?$ $3180074\div56=?$

11. $178464\div16=?$ $385524\div36=?$ $9516783\div64=?$

12. A man bought two farms, one containing 175 acres at \$28 per acre, and the other containing 320 acres at \$37 per acre. What was the cost of both farms?

13. A dealer bought 127 books at 34¢ each, and 97 books at 47¢ each, and sold all of them at 40¢ each. How many cents did he gain?

MULTIPLICATION OF DECIMALS

Multiply 87.7 by 2.03.

$$\begin{array}{r} 87.7 \\ 2.03 \\ \hline 2.631 \\ 175.4 \\ \hline 178.031 \end{array}$$

This result may be proved to be correct by reducing both factors to common fractions

$$87.7 \times 2.03 = \tfrac{877}{10} \times \tfrac{203}{100} = \tfrac{178031}{1000} = 178.031.$$

The number of ciphers in the denominator of the product is equal to the sum of the numbers of ciphers in the denominator of each factor, and the number of ciphers is equal to the number of decimal places; therefore, the number of decimal places in the product must be equal to the sum of the number of decimal places in both factors.

Multiply .0071 by .012.

$$\begin{array}{r} .0071 \\ .012 \\ \hline .0000852 \end{array}$$

Or, $.0071 \times .012 = \tfrac{71}{10000} \times \tfrac{12}{1000} = \tfrac{852}{10000000} = .0000852.$

Here, since there are seven ciphers in the denominator, seven decimal places must be marked off in the decimal fraction. Since there are but three figures indicating units of value, four ciphers must be prefixed to make up the required number of places.

In multiplying decimals we point off in the product as many decimal places as the sum of the decimal places in both multiplicand and multiplier.

Find the products:

1.	2.	3.
a. .012 × .4.	*a.* 150 × .05.	*a.* 25.6 × 25.
b. 101 × .001.	*b.* .925 × 2.5.	*b.* 140.03 × .007.
c. .1 × 20.	*c.* .240 × .003.	*c.* .0029 × 2.51.
d. .032 × 80.	*d.* .024 × 300.	*d.* 1.011 × 10.11.
e. 50 × .25.	*e.* 1.1 × .1.	*e.* .59 × 83.
f. .018 × .03.	*f.* 100 × .001.	*f.* 8.7651 × 30.
g. .625 × .05.	*g.* 400 × .04.	*g.* .0025 × 1240.
h. .001 × 1.01.	*h.* 732 × .625.	*h.* 100.001 × 100.1.
i. 30.5 × 7.5.	*i.* .769 × 2900.	*i.* 3.10459 × 3.14159.
j. 47.001 × .304.	*j.* 1.153825 × .404.	*j.* 34.736 × .00073.

4. In 1 rod there are 16.5 ft. How many feet are there in 41.3 rd.? In 48.75 rd.?

5. In 1 barrel there are 31.5 gal. How many gallons in 65.25 bbl.? In 100 bbl.?

6. In 1 square rod there are 30.25 square yards. How many square yards are there in 26.05 square rods?

7. In 1 square rod are 272.25 sq. ft. How many square feet are in 160 sq. rd.? in 500 sq. rd.?

Find the products:

8.	9.
a. 17.406 × 3.7 × .01.	*a.* 5.4 × 7.3 × .5.
b. 10.3 × .011 × 15.99.	*b.* 4.93 × 13.637 × 12.
c. 17.39 × .00067 × 13.98.	*c.* 147.6984 × .00697.
d. .2 × .2 × .2.	*d.* 3.7 × 4.73 × .001.
e. 30.92 × 2.07 × 51.	*e.* 1000 × .2 × .05.
f. 1.01 × 10.1 × 101 × 101.	*f.* .05 × .005 × .5 × 500.

10. What is the area of a parallelogram, whose base is 176.19 ft. and altitude 142.38 ft.?

DIVISION OF DECIMALS

1. Divide .285 by 1.5.

$$\begin{array}{r} 1.5\overset{\vee}{)}.285(.19 \\ 15 \\ \hline 135 \\ 135 \\ \hline \end{array}$$

Since the dividend is the product of the divisor and quotient, the number of decimal places in the quotient must equal the excess of the number of decimal places in the dividend over the number of places in the divisor.

The carat $\vee$ shows the excess in the problem.

The correctness of the work may be tested by reducing the decimals to common fractions and dividing.

$.285 = \frac{285}{1000}$

$1.5 = \frac{15}{10}$

$$\frac{285}{1000} \div \frac{15}{10} = \frac{\overset{19}{\cancel{285}}}{\underset{100}{\cancel{1000}}} \times \frac{\cancel{10}}{\cancel{15}_{3}} = \frac{19}{100} = .19$$

2. Divide 114.375 by 1.525.

$$\begin{array}{r} 1.525)114.375\overset{\vee}{)}(75 \\ 106\ 75 \\ \hline 7\ 625 \\ 7\ 625 \\ \hline \end{array}$$

Why are there no decimal places in the quotient?

Or, $114.375 \div 1.525 = \frac{114375}{\cancel{1000}} \times \frac{\cancel{1000}}{1525} = \frac{114375}{1525} = 75$

3. Divide .175 by 40.

$$\begin{array}{r} 40\overset{\vee}{)}.175000(4375 \\ 160 \\ \hline 150 \\ 120 \\ \hline 300 \\ 280 \\ \hline 200 \\ 200 \\ \hline \end{array} \qquad .04375$$

In decimals a smaller number may be divided by a larger.

Why is it necessary to add the cipher before the figure 4 in the quotient?

4. Divide:

a. 228.75 by 7.5.
b. 1.96042 by 140.03.
c. .3466 by .0054.
d. .001255 by 1004.
e. 2.568 by 15.
f. 7.1125 by 6.5.
g. .0625 by 1.75.
h. 3.1 by .0125.
i. .0077 by .875.
j. 31.5 by .126.
k. 3217 by .0625.
l. 4.63638 by 81.34.
m. .429408 by 59.64.
n. 12.6 by .0012.
o. 3.1416 by .7854.
p. 2230.1 by .769.
q. .00456 by .1368.
r. 12.65 by .01004.
s. 3.1146 by .036.
t. .0495 by .03888.
u. 6.1325 by 175.
v. 4.8 by .0032.
w. 4.96723 by 23.9.
x. 5.2 by .32.
y. .03217 by 6250.
z. 15.4546 by .019.
aa. 2147.04 by .036.
bb. .065341 by .000475.

5. Divide 1 by .1; by .01; and by .0001.

By means of the decimal notation any number large or small may be divided by any other number large or small. But often the quotients are very, very long; and often in them the figures in the quotients repeat themselves after certain stages in the division.

6. 175. divided by 8155 may be carried to very, very many places in the quotient. Carry the figures of the quotient to ten decimal places.

7. $10 \div 3 = 3.333333$, etc. $10 \div 6 = 1.66666$, etc.

The $3.333+$ may be indicated by a dot over 3.

The $1.666+$ may be indicated by a dot over 1.6.

8. Divide 1. by 13; by 17; and by 9.

9. The area of a parallelogram is 8295 sq. ft. and its base is 905.15 ft. What is its altitude?

DIVISION OF DECIMALS

1. Find the quotients to three places of decimals:

a. .817 ÷ .9147.	*i*. 61.123 ÷ .0146.
b. .213 ÷ 91.614.	*j*. 9.1534 ÷ .000716.
c. 9378.92 ÷ 9.789.	*k*. .0467 ÷ .01471.
d. .793 ÷ .49.	*l*. 918 ÷ 914.71.
e. 8.8 ÷ .0641.	*m*. 43 ÷ .0006947.
f. 7147.12 ÷ 1127.	*n*. 967.104 ÷ 12.046.
g. 78.1 ÷ 1.071.	*o*. 91.671 ÷ .000916.
h. 91.142 ÷ 7.8.	*p*. 3.26 ÷ .111.

2. Divide:

a. 3.012 by .0006. *b*. 293916.669 by 541.283.

c. 130.4 by .0004. *d*. by 4. *e*. 46.634205 by 4807.65.

f. 1.69 by 1.3. *g*. by .13; *h*. by 13; *i*. and by .013.

j. .00281 by 1.405. *k*. by 1405. *l*. and by .001405.

m. 72.36 by 36; *n*. by .0036. *o*. .003 by 1.6.

p. 6725402.3544 by 7089. *q*. and by .7089.

3. A grocer paid $9.45 for a barrel of brown sugar, weighing 315 lb. What was the cost per pound?

4. At the price of $5.75 a ton for coal, how many tons does a bill for $155.25 represent?

5. At the price of $.59125 per bushel, how many bushels does a sale amounting to $4730 represent?

6. At the price of $22.25 per ton, what number of tons does a sale amounting to $77.875 represent?

7. $10.5 \times .05 \div 2.5 = ?$ 8. $25.16 \div .08 \times .3 = ?$

9. $\$.384 \times 7.5 \div .5 = ?$ 10. $\$.125 \times .10 \times 6 = ?$

11. Draw to the scale of $\frac{1}{8}$ in. to 1 ft. a parallelogram, whose area is 100 sq. ft. and whose altitude is $16.\dot{6}$ ft.

REVIEW

1. How many coats can be made from 34 yd. of cloth, allowing 4.25 yd. for each coat?

2. How many cubic feet of air are there in a schoolroom 18 ft. × 13 ft. 4 in. × 10 ft. 6 in.

3. At $25.125 per T., what will 127.045 T. of salt cost?

4. How many cords are there in a pile of cordwood 64 ft. long by 6 ft. high?

5. At $0.125 per pound, how many pounds of prunes can be bought for $2.25?

6. How many cubic yards are there in a rectangular excavation for a cellar 24 ft. × 18 ft. × 5 ft.?

7. A dealer bought 420.25 tons of potash for $35.08 a ton. He sold 196.175 tons for $37.60 a ton, and the rest for $36.125 a ton. How much did he make?

8. How long will it take a railroad train, traveling 26.18 miles an hour, to travel 366.52 miles?

9. How many tons of soft coal can be put in a rectangular bin 7 ft. long, 4 ft. wide, and 3 ft. deep?
(1 ton of soft coal contains 42 cu. ft.)

10. What will 27.5 bu. of potatoes cost at $.875 per bu.?

11. A farmer had 23.478 A. in one field, 29.38 A. in a second field, 18.076 A. in a third field, 0.875 A. as garden, and 1.305 A. in an orchard. The rest of his farm of 100 A. was in woodland. How many A. of woodland had he?

12. A man having nine tenths of an acre of land, sold nineteen thousandths of an acre. How much was left?

13. How many tons of hard coal will a rectangular bin 9 ft. long, 5 ft. 6 in. wide, and 4 ft. deep hold?
(1 ton of hard coal contains 33 cu. ft.)

REVIEW

1. What is the ratio of 18 cu. ft. to 3 cu. yd. ?

2. There are 5.5 yards in one rod, and 1760 yards in a mile. How many rods are there in a mile ?

3. How many feet are there in a mile ?

4. *a.* $159984 \div 144 = ?$ *b.* $33768 \div 168 = ?$
   c. $58420 \div 135 = ?$ *d.* $49102 \div 81 = ?$
   e. $687310 \div 132 = ?$ *f.* $47631 \div 144 = ?$
   g. $876312 \div 240 = ?$ *h.* $76543 \div 108 = ?$

5. What is the ratio of a foot to a mile ?

6. At \$0.21 a pound, how many pounds of Swiss cheese can be bought for \$1613.22 ?

7. *a.* Find the circumference of a circle whose diameter is 6.8 ft. The circumference = diameter × 3.1416.

b. Find the diameter of a circle whose circumference is 20 in. The diameter of a circle = circumference ÷ 3.1416.

8. How much wire will be required to make a hoop 2 ft. 6 in. in diameter, allowing 2 in. for the joining ?

9. How many times would such a hoop turn in $\frac{1}{2}$ mi. ?

10. How many tons are there in 3 cars of coal, the first car containing 18.375 tons, the second car 41.625 tons, and the third car 35.5 tons ?

11. If a man earns \$180 in a month of 25 da., how much does he earn in 1 da. ?

12. How long would it take to travel 30.2375 miles at the rate of 8.85 miles per hour ?

13. The prices of 3 kinds of lace are \$2.22 a yard, \$0.98 a yard, and \$1.16 a yard. How many yards of each kind can one buy for \$242.97, if all the money is spent for that one kind ?

14. What is the cost of $\frac{1}{2}$ lb. of bird seed at 4¢ per oz. ?

15. How many ounces are there in 2 lb. of sugar? in 2 lb. of silver?

16. What does the decimal point tell us?

17. If pepper is 40¢ per lb., what will 4 oz. cost?

18. Washington was born 1732 A.D. and lived 67 years. In what year did he die?

19. Babylon is said to have been built in the year 3100 B.C. How long was it to the settlement of Jamestown, Va., 1607 A.D.?

20. A republic was founded in ancient Greece in the year 510 B.C. How long was it from that year to the Declaration of American Independence in 1776 A.D.?

21. X sold a load of wood for $18. He spent $4 for groceries and $6 for meat. How many dollars had he left?

22. A grocer bought 150 dozen eggs, and sold 125 dozen. How many eggs were there left?

23. How many are 29 less 11? 29 less 13? 19 less 5? 19 less 17? 29 less 9? 29 less 16?

24. I bought a bicycle for $45, and sold it for $29. How much did I gain or lose?

25. I sold a watch for $62, which cost me $85. How much did I gain or lose?

26. I walked 27 miles in 1 day and my brother walked 39 miles. How much farther did he walk?

27. A man had $110 and paid $35 to one person and $40 to another person. How many dollars had he left?

28. How many are $80 - 9 - 6 - 7 - 1 - 2 - 4 - 5 - 6$?

29. How many are $9 + 3 + 4 + 9 + 7 + 6 - 7 - 4 - 3 - 7 - 9 - 1$?

30. William had 67¢; he spent 5¢ for oranges and 25¢ for a flag. How many cents had he left?

ADDITION

1. The United States military forces were distributed as follows in 1899 (Dec. 10):

In the United States	34,299 men,
In Porto Rico	3,363 men,
In Cuba	11,187 men,
In Philippine Islands	32,315 men,
En route to Philippine Islands	17,099 men,
In Alaska	499 men,
In Hawaiian Islands	466 men.

How many soldiers were there in all?

2. In 1899 there arrived in the United States from:

Austria-Hungary	62,491 immigrants,
Germany	17,476 immigrants,
Italy	77,419 immigrants,
Russia	60,982 immigrants,
Sweden	12,797 immigrants,
United Kingdom	45,181 immigrants.

What was the total number of immigrants from these countries?

3. In the year 1899 the United States government expended the following sums of money:

War Dept.	$229,841,254.
Navy Dept.	63,942,104.
Pensions	139,384,929.
Miscellaneous	119,191,255.
Ordinary Expenses	565,175,254.
Interest	39,896,925.

What was the total amount expended?

WRITTEN AND ORAL QUESTIONS

Many problems may be made oral by using sufficiently easy numbers. Compare the following:

1. *a.* How many cord feet are there in 17 cd. of wood?

1. *b.* ORAL. How many cord feet are there in 5 cd.?

2. *a.* Mary had 85% in reading, 90% in arithmetic, 75% in nature-study, and 80% in drawing. If reading and arithmetic each count 2 points, and nature-study and drawing each 1 point, what was her average mark?

2. *b.* ORAL. Mary had 9 in reading, 9 in arithmetic, 8 in nature-study, and 8 in drawing. If the first two studies each counted two points, and the second two each one point, what was her average?

3. *a.* Find x. $x : 1250 = 10 : 50.$

3. *b.* ORAL. Find x. $x : 100 = 8 : 16.$

4. *a.* What per cent of 31,000 bushels is 425 bushels?

4. *b.* ORAL. What per cent of 25 is 2.5?

5. A cubic foot of water weighs 62.5 lb. What is the weight of water in a tank 10 ft. × 5 ft. × 2 ft.?

6. A hand is 4 in. What is the height in ft. and in. of a race horse measuring $16\frac{1}{2}$ hands over the front feet?

7. There are 7.5 gal. in 1 cu. ft. A tank measures 3 ft. × 5 ft. × 20 ft. How many gal. does it hold?

8. A barrel holds 4.2 cu. ft. How many bbl. are needed to hold 158 gal. of water?

9. How many more cu. in. are in 1 bu. than in 1 cu. ft.?

10. One horse ate 10 bu. of oats in a month; another ate 7 bu. of corn. Which ate the greater weight of grain? 1 bu. of oats weighs 32 lb. 1 bu. of corn 56 lb.

11. How many quarts per day was this for each?

COMMON FRACTIONS

UNIT NUMERATORS

A fraction is one or more of the equal parts of anything.

1. What is $\frac{1}{2}$ of 4? 8? 10? 14? 16? 24? 30?
2. What is $\frac{1}{3}$ of 3? 6? 9? 12? 15? 18? 21?
3. What is $\frac{1}{4}$ of 4? 8? 12? 16? 20? 24? 36?
4. What is $\frac{1}{5}$ of 5? 10? 15? 20? 25? 30? 45?
5. What is $\frac{1}{6}$ of 6? 12? 18? 24? 30? 36? 48?
6. What is $\frac{1}{7}$ of 7? 14? 21? 28? 42? 70? 77?
7. What is $\frac{1}{8}$ of 8? 16? 24? 32? 40? 56? 80?
8. What is $\frac{1}{9}$ of 9? 18? 27? 63? 99? 180? 450?
9. What is $\frac{1}{10}$ of 10? 20? 30? 40? 60? 100? 1000?
10. What is $\frac{1}{11}$ of 11? 22? 33? 44? 55? 121? 132?
11. What is $\frac{1}{12}$ of 24? 36? 60? 72? 96? 144? 288?

We must know the **multiplication tables** accurately in order to answer questions in common fractions.

MULTIPLICATION BY FRACTIONS

12. At 12¢ apiece, what will $2\frac{1}{2}$ pies cost?
13. At $6 a barrel, what will $3\frac{1}{3}$ barrels of flour cost?
14. At $3 a yard, what will $4\frac{2}{3}$ yards of cloth cost?
15. If 3 bushel of barley cost $1, what part of a dollar will 1 bushel cost? What part of a dollar will 2 bushels cost?
16. What is $\frac{1}{3}$ of 1? $\frac{2}{3}$ of 1? of 2? of 3?
17. What is $15 \times 2\frac{1}{3}$? $16 \times 2\frac{1}{4}$?
18. At $1\frac{1}{4}$ each, what is the cost of one dozen baseballs?
19. At $5\frac{1}{2}$¢ a pound, what is the cost of 10 lb. of sugar? of 100 lb.? of 2 cwt.?

QUESTIONS

1. If 3 bbl. of apples cost \$4, what will 1 bbl. cost?

2. If 3 yards of cloth cost \$2, what part of a dollar will 1 yard cost? How much will 2 yards cost?

3. How many barrels of vinegar, at \$7 a barrel, can one buy for \$15? for \$16? for \$20?

4. How many yards of silk, at \$6 a yard, can one buy for \$7? for \$9? for \$10? for \$14?

5. What number is $\frac{4}{8}$ of 8? What is $\frac{5}{8}$ of 8? $\frac{7}{8}$ of 8?

6. If a melon cost 16¢, what is $\frac{1}{8}$ of the melon worth? What are $\frac{2}{8}$ of it worth? $\frac{5}{8}$? $\frac{7}{8}$?

7. How many quarts of milk, at 6¢ a quart, can one buy for 16¢? for 17¢? for 18¢?

8. If gingham is 9¢ a yard, how many yards can one buy for 10¢? for 17¢? for 18¢? for 20¢?

9. If apples cost 10¢ a quart, what is $\frac{1}{10}$ of a quart worth? $\frac{3}{10}$? $\frac{5}{10}$? $\frac{7}{10}$?

10. At \$10 a yard, how much velvet can one buy for \$11? for \$14? for \$19?

11. If it requires 70 hours to perform a piece of work, in how many days can it be done by working 7 hours a day? by working 8 hours a day? by working 9 hours a day? by working 10 hours a day?

12. How many eighths of a cent are there in 24¢?

13. How many ninths of an orange are there in 32 oranges?

14. If a yard of cloth cost \$3, how many yards can one buy for \$4? How many for \$5?

15. What is the area of a rectangle whose base is $3\frac{1}{2}$ miles and whose altitude is 4 miles?

Draw this on the blackboard: scale 3 in. = 1 mi.

COMMON FRACTIONS

The number which shows into how many equal parts the unit has been divided is called the **denominator** of the fraction; and the number which shows how many times one of these parts has been taken is called the **numerator** of the fraction. The numerator and the denominator are called the **terms** of the fraction.

We express a fraction in figures by writing the numerator above the denominator and drawing a bar between them; the fraction four fifths, which has 4 for its numerator and 5 for its denominator, is written $\frac{4}{5}$; eleven nineteenths is written $\frac{11}{19}$; thirty-six forty-eighths is written $\frac{36}{48}$.

A fraction expresses an unperformed division, in which the numerator is the dividend and the denominator is the divisor.

A **proper fraction** indicates a quantity less than one whole thing. The numerator of a *proper* fraction is always smaller than the denominator.

$\frac{3}{8}$, $\frac{5}{9}$, $\frac{23}{24}$, $\frac{99}{100}$, are proper fractions. Why?

1. Give other proper fractions.

An **improper fraction** is one whose numerator is not less than its denominator. The value of an *improper* fraction is never less than a unit.

$\frac{6}{6}$, $\frac{13}{12}$, $\frac{100}{24}$, are improper fractions. Why?

2. Give other improper fractions.

A **mixed number** is an integer and a fraction united. $12\frac{7}{8}$, $39\frac{3}{17}$, $105\frac{7}{12}$, are mixed numbers. Why?

3. Give others, using facts in weights and measures.

A **simple fraction** is one whose numerator and denominator are each single integral numbers.

$\frac{3}{4}$, $\frac{10}{7}$, are simple fractions.

4. Give other simple fractions.

PRINCIPLES OF FRACTIONS

The **value** of the fraction is the quotient obtained by dividing the numerator by the denominator.

I. $\frac{1}{3} \times 2 = \frac{2}{3}$ $\frac{3}{5} \times 4 = \frac{12}{5}$ $\frac{7}{50} \times 6 = \frac{42}{50}$

Multiplying the numerator of a fraction by any number multiplies the fraction by that number.

II. $\frac{1}{2 \times 4} = \frac{1}{8}$ $\frac{1}{5 \times 6} = \frac{1}{30}$ $\frac{7}{50 \times 2} = \frac{7}{100}$

Multiplying the denominator of a fraction by any number divides the fraction by that number.

III. $\frac{1 \times 2}{2 \times 2} = \frac{2}{4} = \frac{1}{2}$ $\frac{2}{5} \times \frac{5}{5} = \frac{10}{25} = \frac{2}{5}$

Multiplying both numerator and denominator of a fraction by the same number does not change the value of the fraction.

IV. $\frac{2 \div 2}{5} = \frac{1}{5}$ $\frac{10 \div 5}{25} = \frac{2}{25}$ $\frac{39 \div 13}{50} = \frac{3}{50}$

Dividing the numerator of a fraction by any number divides the fraction by that number.

V. $\frac{10}{25 \div 5} = \frac{10}{5}$ $\frac{7}{50 \div 5} = \frac{7}{10}$ $\frac{8}{100 \div 10} = \frac{8}{10}$

Dividing the denominator of a fraction by any number multiplies the fraction by that number.

VI. $\frac{10 \div 5}{25 \div 5} = \frac{2}{5} = \frac{10}{25}$ $\frac{24 \div 4}{100 \div 4} = \frac{6}{25} = \frac{24}{100}$

Dividing both numerator and denominator of a fraction by the same number does not affect its value.

VII. $\frac{3}{5+1} = \frac{3}{6}$ $\frac{3}{5-1} = \frac{3}{4}$ $\frac{3+1}{5} = \frac{4}{5}$ $\frac{3-1}{5} = \frac{2}{5}$

Increasing the numerator produces an increase in the value of the fraction; but increasing the denominator decreases its value. Decreasing the numerator decreases the value of the fraction; but decreasing the denominator increases its value.

ANALYSIS OF FRACTIONS

1. In the fraction $\frac{6}{11}$ 11 is the denominator, and shows into how many parts the magnitude has been divided. The parts are elevenths. 6 is the numerator and shows how many parts have been taken. The numbers 6 and 11 are the terms of the fraction. The fraction $\frac{6}{11}$ is an expression of unperformed division, 6 being the dividend and 11 the divisor. It is read six elevenths, and represents 6 of 11 equal parts of 1 or $\frac{1}{11}$ of 6.

2. Analyze the following fractions:

a. $\frac{2}{9}$.	*b.* $\frac{5}{7}$.	*c.* $\frac{8}{12}$.	*d.* $\frac{6}{10}$.	*e.* $\frac{7}{8}$.	*f.* $\frac{14}{15}$.
g. $\frac{20}{21}$.	*h.* $\frac{32}{30}$.	*i.* $\frac{80}{80}$.	*j.* $\frac{44}{45}$.	*k.* $\frac{90}{91}$.	*l.* $\frac{25}{24}$.
m. $\frac{66}{66}$.	*n.* $\frac{212}{213}$.	*o.* $\frac{213}{212}$.	*p.* $\frac{13}{141}$.	*q.* $\frac{27}{20}$.	*r.* $\frac{48}{81}$.
s. $\frac{89}{88}$.	*t.* $\frac{120}{10}$.	*u.* $\frac{224}{220}$.	*v.* $\frac{300}{100}$.	*w.* $\frac{85}{200}$.	*x.* $\frac{775}{1000}$.

3. Express in figures:

a. Three fourths. *b.* Eight ninths. *c.* Five sixths.
d. Seven twelfths. *e.* Twelve seventeenths.
f. Four twenty-firsts. *g.* Eleven forty-fourths.
h. Thirty-three thirty-fifths. *i.* Seventy-one eighty-firsts.
j. One hundred twelve ninety-fifths.
k. Two hundred five hundredths.

A **complex fraction** is a fraction containing a fraction or a mixed number in either numerator or denominator, or in both numerator and denominator.

$$\frac{2}{\frac{3}{7}}, \quad \frac{3}{5\frac{4}{9}}, \quad \frac{2\frac{1}{2}}{4\frac{3}{4}}, \quad \frac{3\frac{4}{5}-2\frac{2}{3}}{3\frac{4}{5}+2\frac{2}{3}}$$

A compound fraction is a fraction of a fraction:

$$\tfrac{3}{5} \text{ of } \tfrac{7}{8}; \quad \tfrac{13}{15} \times \tfrac{18}{21}; \quad \tfrac{6}{7} \div \tfrac{14}{30}.$$

Both complex and compound fractions may be reduced to simple fractions by division and cancellation, as we shall see later.

REDUCTION OF FRACTIONS

Reduction of fractions changes their forms without altering their values. We may reduce whole numbers to fractions of the same values.

1. Reduce \$5 to quarters.

4 quarters
5
20 quarters

Since in \$1 there are 4 quarters, in \$5 there are 5 times 4 quarters = 20 quarters.

or $\frac{5}{1} \times \frac{4}{4} = \frac{20}{4}$.

2. Change $\frac{7}{9}$ to 18ths.

As the denominator is to be 18, and since $18 = 9 \times 2$, we multiply both terms by 2. $\frac{14}{18}$ indicates equal parts half as large as those in $\frac{7}{9}$, but in $\frac{14}{18}$ twice as many parts are taken as in $\frac{7}{9}$.

$\frac{7}{9} \times \frac{2}{2} = \frac{14}{18}$.

3. Change:

a. $\frac{8}{13}$ to 65ths. *c.* $\frac{11}{15}$ to 90ths. *e.* $\frac{9}{17}$ to 136ths.

b. $\frac{5}{6}$ to 72ds. *d.* $\frac{4}{9}$ to 81sts. *f.* $\frac{3}{7}$ to 63ds.

QUESTIONS

1. Write five proper fractions; five improper fractions; five mixed numbers.

2. Write five simple fractions; five complex fractions; five compound fractions.

3. Which is the larger amount:

$\frac{5}{6}$ of anything or $\frac{5}{24}$ of it? $\frac{9}{90}$ or $\frac{90}{100}$ of anything?

$\frac{15}{12}$ of anything or $\frac{15}{16}$ of it? $\frac{7}{8}$ pk. or $\frac{15}{16}$ pk?

Give the reason in each case, or show by drawing.

4. Which is larger: $\frac{1}{2}$ of $\frac{1}{4}$ or $\frac{1}{4} \times 2$? By how much?

REDUCING INTEGERS TO FRACTIONS

1. Reduce 123 to a fraction with 20 as its denominator.

$$123 = \tfrac{123}{1}, \qquad \tfrac{123}{1} \times \tfrac{20}{20} = \tfrac{2460}{20}$$

2. Change 17 to a fraction with 29 as its denominator.

$$17 = \tfrac{17}{1}, \qquad \tfrac{17}{1} \times \tfrac{29}{29} = \tfrac{493}{29}$$

3. Reduce 7, 9, 27, and 40 to elevenths.

4. Reduce 2, 207, 440, and 9 to fractions with 109 as denominator.

5. Reduce 22, 47, 69, and 100 to ninety-thirds.

6. Reduce 217, 613, 927, and 4 to thirteenths.

REDUCING MIXED NUMBERS TO FRACTIONS

7. Reduce $7\frac{4}{9}$ to an improper fraction.

Since in 1 there are 9 ninths, in 7 there are 7 times 9 ninths. $7 \times \frac{9}{9} = \frac{63}{9}$, $\frac{63}{9} + \frac{4}{9} = \frac{67}{9}$.

8. Change $167\frac{14}{19}$ to an improper fraction.

$167 \times 19 = 3173$

$\frac{3173}{19} + \frac{14}{19} = \frac{3187}{19}$

$167\frac{14}{19} = \frac{3187}{19}$

We multiply the integer by the number in the denominator of the fraction, and write the product as the numerator of an improper fraction. To this improper fraction we add the fraction in the mixed number.

9. Reduce to improper fractions:

a. $73\frac{4}{19}$.	*f.* $6\frac{4}{13}$.	*k.* $\$4\frac{9}{10}$.	*p.* $5\frac{75}{5280}$ mi.
b. $18\frac{4}{11}$.	*g.* $7\frac{2}{5}$.	*l.* $18\frac{3}{8}$ pk.	*q.* $82\frac{11}{16}$ hr.
c. $16\frac{3}{7}$.	*h.* $9\frac{3}{8}$.	*m.* $10\frac{3}{4}$ cd.	*r.* $1\frac{300}{1728}$ cu. ft.
d. $14\frac{3}{10}$.	*i.* $16\frac{2}{3}$.	*n.* $18\frac{19}{20}$ sq. ft.	*s.* $36\frac{5}{24}$ da.
e. $71\frac{3}{5}$.	*j.* $25\frac{19}{72}$.	*o.* $160\frac{4}{5}$ A.	*t.* $10\frac{53}{144}$ G. gr.

IMPROPER FRACTIONS

1. Reduce 29 to a fraction having 12 for its denominator.

2. Reduce 243 to a fraction having 3 for its denominator.

3. Reduce 7, 23, and 101 to fractions having 18 for denominator.

4. Reduce to improper fractions:

a. $39\frac{7}{8}$. *d.* $676\frac{37}{51}$. *g.* $847\frac{36}{175}$. *j.* $375\frac{3}{4}$. *m.* $1789\frac{5}{9}$.
b. $112\frac{9}{10}$. *e.* $464\frac{19}{63}$. *h.* $675\frac{187}{200}$. *k.* $375\frac{94}{99}$. *n.* $125\frac{6}{7}$.
c. $427\frac{11}{24}$. *f.* $367\frac{9}{104}$. *i.* $149\frac{5}{9}$. *l.* $4834\frac{57}{95}$. *o.* $10\frac{75}{100}$.

MIXED NUMBERS

5. Reduce $\frac{117}{5}$ to a mixed number.

$\frac{117}{5} = 117 \div 5 = 23\frac{2}{5}$

$5\overline{)117}$

$23\frac{2}{5}$

Since 5 fifths = 1, 117 fifths equal as many 1's as 5 is contained times in 117. 5 is found in 117 $23\frac{2}{5}$ times.

6. Change $\frac{648}{45}$ to a mixed number.

$\frac{648}{45} = 648 \div 45 = 14\frac{18}{45} = 14\frac{2}{5}$

We divide the numerator by the denominator.

7. Reduce to whole or mixed numbers:

a. $\frac{71}{4}$. *f.* $\frac{200}{7}$. *k.* $\frac{4547}{48}$. *p.* $\frac{284}{32}$. *u.* $\frac{96}{8}$.
b. $\frac{1694}{11}$. *g.* $\frac{178}{87}$. *l.* $\frac{123}{13}$. *q.* $\frac{2964}{25}$. *v.* $\frac{1000}{7}$.
c. $\frac{1795}{17}$. *h.* $\frac{1462}{95}$. *m.* $\frac{279}{60}$. *r.* $\frac{4198}{7}$. *w.* $\frac{743}{81}$.
d. $\frac{6262}{44}$. *i.* $\frac{1279}{125}$. *n.* $\frac{407}{13}$. *s.* $\frac{29817}{365}$ *x.* $\frac{805}{5}$.
e. $\frac{714}{19}$. *j.* $\frac{113}{17}$. *o.* $\frac{60}{25}$. *t.* $\frac{539}{89}$. *y.* $\frac{1728}{12}$.

8. Reduce to mixed numbers:

a. $\frac{100}{8}$ bu. *b.* $\frac{200}{9}$ sq. ft. *c.* $\frac{3000}{640}$ A. *d.* $\frac{500}{231}$ cu. in.
e. $\frac{183}{6}$ doz. *f.* $\frac{129}{8}$ cd. *g.* $\frac{75}{4}$ gal. *h.* $\frac{1000}{3}$ rd.

REDUCTION OF DECIMALS TO COMMON FRACTIONS

1. Reduce .95 to a common fraction at its lowest terms.

$$.95 = \frac{95}{100} = \frac{19}{20}$$

We divide 95 and 100 by the common factor 5.

2. Reduce .0125 to a common fraction at its lowest terms.

$$.0125 = \frac{125}{10000} = \frac{25}{2000} = \frac{5}{400} = \frac{1}{80}$$

We divide 125 and 10000 by 5, 5, and 5, the common factors of the numerator and denominator.

3. Reduce to a common fraction at lowest terms:

a. .0025.	*d*. .204.	*g*. .435.	*j*. .124.
b. .0065.	*e*. .05.	*h*. .575.	*k*. .486.
c. .375.	*f*. .225.	*i*. .012.	*l*. .225.

It is very seldom helpful to change a decimal to a common fraction with a denominator larger than one hundred. It is more often helpful to change a common fraction to a decimal than to change a decimal to a common fraction.

RECITE

REVIEW

4. What is the ratio of 1 to .25? of .25 to 1?

5. What is the ratio of 1 to .3? of .3 to 1?

6. What is the ratio of 1.25 to 10? of 10 to 1.25?

7. What is the difference between

a. \$575 and \$.50? *b*. \$3.75 and \$1.375?

c. \$1 and \$.66+? *d*. \$2.50 and \$1.166+?

8. What is the sum of \$12.33+ and \$12.66+?

9. Make drawings to illustrate **4**, **5**, and **6**.

REDUCTION OF COMMON FRACTIONS TO DECIMALS

1. Change $\frac{3}{5}$ into a decimal.

$$5)3.0$$
$$.6$$

We divide the numerator by the denominator.

There is one decimal place in the dividend, and none in the divisor; therefore, there is one decimal place in the quotient.

2. Reduce $\frac{5}{16}$ to a decimal.

$$16)5.0000(.3125$$
$$\underline{48}$$
$$20$$
$$\underline{16}$$
$$40$$
$$\underline{32}$$
$$80$$
$$\underline{80}$$

The numerator is treated as a whole number; any annexed ciphers are treated as indicating decimal fraction places.

We point off the quotient as always in the division of decimals.

3. Change to decimals:

a. $\frac{1}{2}$. *c*. $\frac{17}{20}$. *e*. $\frac{5}{8}$. *g*. $\frac{7}{64}$. *i*. $\frac{1}{75}$.

b. $\frac{4}{5}$. *d*. $\frac{3}{4}$. *f*. $\frac{5}{16}$. *h*. $\frac{3}{50}$. *j*. $\frac{84}{138}$.

4. Change $\frac{2}{3}$ to a decimal.

$$3)2.00$$
$$.66\tfrac{2}{3}$$

We may write the answer .66+ or $.\dot{6}$ or $.6\frac{2}{3}$.

A common fraction whose denominator contains any factor other than 2 or 5 cannot be reduced to a pure decimal without remainder, unless its own numerator contains that other factor. The decimals with remainders in the common fraction form, or of that value, are "approximate" decimals.

REDUCTION OF DECIMALS

1. Change to decimals:

a. $\frac{1}{3}$.	*f.* $\frac{3}{8}$.	*k.* $\frac{8}{9}$.	*p.* $\frac{13}{22}$.	*u.* $\frac{1}{66}$.
b. $\frac{1}{6}$.	*g.* $\frac{5}{12}$.	*l.* $\frac{11}{12}$.	*q.* $\frac{11}{36}$.	*v.* $\frac{1}{133}$.
c. $\frac{5}{6}$.	*h.* $\frac{7}{8}$.	*m.* $\frac{4}{9}$.	*r.* $\frac{3}{50}$.	*w.* $\frac{1}{200}$.
d. $\frac{1}{12}$.	*i.* $\frac{5}{18}$.	*n.* $\frac{5}{11}$.	*s.* $\frac{7}{60}$.	*x.* $\frac{1}{500}$.
e. $\frac{5}{16}$.	*j.* $\frac{7}{24}$.	*o.* $\frac{8}{15}$.	*t.* $\frac{2}{75}$.	*y.* $\frac{1}{2500}$.

2. Change to decimals, carrying out to not over five decimal places:

a. $\frac{6}{7}$. *b.* $\frac{13}{17}$. *c.* $\frac{8}{17}$. *d.* $\frac{3}{13}$. *e.* $\frac{47}{51}$.

STUDY

TABLE OF EQUIVALENTS

I	$\frac{1}{3} = .33+$	IX	$\frac{1}{6} = .166+$
II	$\frac{2}{3} = .66+$	X	$\frac{5}{6} = .833+$
III	$\frac{1}{4} = .25$	XI	$\frac{1}{8} = .125$
IV	$\frac{3}{4} = .75$	XII	$\frac{3}{8} = .375$
V	$\frac{1}{5} = .2$	XIII	$\frac{5}{8} = .625$
VI	$\frac{2}{5} = .4$	XIV	$\frac{7}{8} = .875$
VII	$\frac{3}{5} = .6$	XV	$\frac{1}{12} = .0833+$
VIII	$\frac{4}{5} = .8$	XVI	$\frac{5}{12} = .5833+$

Prove these equalities by reducing the common fractions to decimals. Make drawings to show some of the quantities indicated.

XVII $\frac{1}{2} = .5 = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} = \frac{7}{14} = \frac{8}{16}$.

XVIII $\frac{1}{3} = .33+ = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \frac{6}{18}$.

XIX $\frac{2}{3} = .66+ = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15} = \frac{12}{18}$.

RECITE

REVIEW

3. What is the ratio of 75% of anything to $12\frac{1}{2}$% of it?
4. What is the ratio of .125 to .375?
5. What per cent is 4 of 10? 3 of 10?
6. $87\frac{1}{2}$% of a thing less 50% of it is how many eighths?

REVIEW

1. A dealer sold 4 suits for $25.50 a suit, 12 vests for $2.12½ each, and 3 coats for $7.25 each. How much did he get for all the garments sold?

2. What are 25 webs of cloth, each web containing 872 yards, worth, at 6½¢ a yard?

3. X bought, at 37½¢ per lb., 9 sacks of copper, each sack containing 72 lb. What was the total cost of the copper?

4. Subtract:

a. 30604	*b.* 31024	*c.* 40679	*d.* 70686	*e.* 89706	*f.* 69807
19087	10013	30416	30425	46302	14306
g. 14071	*h.* 10102	*i.* 19006	*j.* 70601	*k.* 11001	*l.* 30608
12618	8016	8709	5908	10101	7909

5. A man willed to his wife $25,000, to his two sons $19,375 each, to his three daughters $7750 each, and to a hospital $3460. What was the sum of the legacies?

6. Add:

a.
792085579
909672916
1251495987650
868253926
92150470279
3234918065933
2672409308
5368542984945
64709365197
3447615994883772
185938976
934172706050896
685675
891076813

7. Copy, point in periods, and read:

b.	*c.*
92837465321	186457900
9739765689079	920749365
31859678009678	283467458729
9412287064571493	814382257
45902768537	8369376914936
45794555672990	21743790051
63749568	847090439
64573594693845	50210706090
5277456789987654	739634721987
49596878397	68598765432
7395084309724606	119647896
29684371989	5846952081638
23678521	79749327965
139324680357	574981672

REVIEW

1. If 1 acre produces 46 bushels of oats, how many bushels will 358 such acres produce?

2. How many pounds of iron are there in the rails of a railroad track 64 miles long, when 1760 yards make a mile, and every yard of iron rail weighs 65 pounds?

3. If the average mill consumed 428 tons of coal annually, what would be the amount of coal consumed annually in a country containing 15,769 mills?

4. In a building there are 75 rooms, and each room has 4 windows. In each window there are 12 panes of glass, and each pane contains 120 square inches. How many square inches of glass are there in the building?

5. The earth, in its annual revolution, moves 19 miles a second. How far does the earth move in 1 week?

6. One Atlantic cable is composed of 7 copper wires, inclosed in gutta-percha, surrounded by 18 bundles of iron wire, each bundle composed of 7 wires. How many wires are there in the cable?

7. A railroad 95 miles long cost $3,300,000. What was the cost per mile?

8. The Ladysmith garrison had a million days' rations when the siege began. How many days would these last 12,000 soldiers?

9. If 22 bricks make 1 cu. ft. of brickwork, how many cu. ft. are there in a wall which requires 100,000 bricks?

10. Find the value of:

a. $\frac{10}{21}$ of $\frac{35}{36}$. *b.* $\frac{3}{4}$ of $\frac{1}{2}$ ft. *c.* $\frac{1}{6}$ of $\frac{3}{5}$ hr.

d. $\frac{15}{44}$ of $\frac{44}{15}$. *e.* $\frac{4}{5}$ of $\frac{15}{16}$ lb. *f.* $\frac{6}{35}$ of $\frac{14}{15}$ T.

g. $\frac{33}{35}$ of $\frac{42}{65}$. *h.* $\frac{5}{12}$ of $\frac{3}{4}$ yd. *i.* $\frac{8}{11}$ of $\frac{20}{21}$ of $\frac{35}{48}$.

11. In a school there are 150 scholars, 60 of whom are girls. How many boys are there in the school?

12. A man bought a lot for $900, and sold it for $1300. How much did he make by the sale?

13. How many pounds of butter at 19¢ per pound would equal the value of a cow, whose price is $47.50?

14. A man owing $208, pays $88. How much money does he still owe?

15. A boy going to market with 80¢, bought 40¢ worth of sugar and 30¢ worth of butter. What coin or coins may he have had left?

16. If a ship sails 250 miles a day, in how many days would it sail 10,000 miles?

17. For $2032 how many yards of silk at $4 a yard can be bought?

18. Divide:

a. $2\frac{1}{4}$ by 7. *b.* $\frac{7}{19}$ by 14. *c.* $\frac{3}{4}$ by 9. *d.* $\frac{8}{33}$ by 4.

19. John Anderson owns $\frac{5}{7}$ of a railroad share worth $117. He divides it equally among his five children. What part of a share does each child receive?

20. To the sum of 6 and 9 add the difference between 11 and 8.

21. To 13 add the sum of 6 and 5, and from the result take 8. Divide the remainder by 4.

22. From the difference between 7 and 2 take the difference between 13 and 11.

23. A man owned $\frac{3}{5}$ of a ship, and sold $\frac{1}{2}$ of his share. What share of the ship did he sell?

24. If a bushel of barley will buy $\frac{7}{10}$ bu. of wheat, how much wheat can be bought for $\frac{1}{4}$ bu. of barley?

25. The circumference of the earth is 25,000 miles; if a railroad train moves at the rate of 1250 miles a day, how many days would it require to go around the earth?

LOWEST TERMS

A fraction is in its lowest terms when its terms are prime to each other. What are the *terms* of a fraction?

1. Reduce $\frac{12}{36}$ to its lowest terms.

$\frac{12}{36} = \frac{1}{3}$

We divide the terms of the fraction by 12, a factor common to them both, and obtain $\frac{1}{3}$.

Since 1 and 3 are prime to each other, the fraction $\frac{1}{3}$ is in its lowest terms.

Since numerator and denominator correspond to dividend and divisor, dividing both by the same number changes only the form of the fraction, the value of the fraction remaining the same.

2. Reduce $\frac{315}{1890}$ to its lowest terms.

2 will not divide 315, since 315 is not an even number.

315 and 1890 divided by 3 give this new fraction:

We cancel out the 3.

105 and 630 divided by 5 give this new fraction:

We cancel out the 5.

21 and 126 divided by 3 give:

We cancel out another 3.

Last, we cancel out the 7.

At its lowest terms $\frac{315}{1890} = \frac{1}{6}$.

When the terms of the fraction to be reduced are large, we cancel in each term the common prime factors, until the terms are prime to each other.

$$\frac{\cancel{315}^{105}}{\cancel{1890}_{630}} = \frac{105}{630}$$

$$\frac{\cancel{105}^{21}}{\cancel{630}_{126}} = \frac{21}{126}$$

$$\frac{\cancel{21}^{7}}{\cancel{126}_{42}} = \frac{7}{42}$$

$$\frac{\cancel{7}^{1}}{\cancel{42}_{6}} = \frac{1}{6}$$

3. Reduce to lowest terms:

a. $\frac{12}{15}$. *b.* $\frac{27}{36}$. *c.* $\frac{144}{864}$. *d.* $\frac{288}{2592}$. *e.* $\frac{275}{440}$. *f.* $\frac{630}{810}$.

4. Reduce to lowest terms:

a. $\frac{18}{24}$. *b.* $\frac{24}{36}$. *c.* $\frac{36}{144}$. *d.* $\frac{63}{81}$. *e.* $\frac{4}{6}$. *f.* $\frac{3}{9}$.
g. $\frac{4}{12}$. *h.* $\frac{7}{21}$. *i.* $\frac{24}{48}$. *j.* $\frac{32}{48}$. *k.* $\frac{60}{96}$. *l.* $\frac{72}{144}$.
m. $\frac{9}{99}$. *n.* $\frac{72}{96}$. *o.* $\frac{132}{144}$. *p.* $\frac{49}{63}$. *q.* $\frac{81}{90}$. *r.* $\frac{144}{168}$.

5. Reduce to lowest terms: WRITE

a. $\frac{587600}{661105}$. *b.* $\frac{315}{405}$. *c.* $\frac{308}{506}$. *d.* $\frac{364}{404}$. *e.* $\frac{728}{808}$.
f. $\frac{1362}{1500}$. *g.* $\frac{81}{486}$. *h.* $\frac{168}{216}$. *i.* $\frac{160}{352}$. *j.* $\frac{144}{208}$.
k. $\frac{120}{225}$. *l.* $\frac{784}{1232}$. *m.* $\frac{2304000}{5376000}$. *n.* $\frac{28}{308}$. *o.* $\frac{1500}{1600}$.
p. $\frac{990}{1000}$. *q.* $\frac{10000}{100000}$. *r.* $\frac{1000}{1000000}$. *s.* $\frac{500}{25000}$. *t.* $\frac{110}{10000}$.

REDUCTION TO A COMMON DENOMINATOR

6. Reduce $\frac{3}{4}$, $\frac{7}{11}$, and $\frac{5}{9}$ to a common denominator.

$3 \times 11 \times 9 = 297 =$ first numerator
$7 \times 4 \times 9 = 252 =$ second numerator
$5 \times 4 \times 11 = 220 =$ third numerator
$4 \times 11 \times 9 = 396 =$ common denominator
$\frac{3}{4} = \frac{297}{396}$, $\frac{7}{11} = \frac{252}{396}$, and $\frac{5}{9} = \frac{220}{396}$

We multiply the numerator of each fraction by the denominators of the other fractions to find the new numerators; and we multiply all the denominators together for the new common denominator.

7. Reduce to equivalent fractions with a common denominator:

a. $\frac{1}{2}, \frac{3}{5}, \frac{4}{7}, \frac{9}{11}$. *b.* $\frac{18}{20}, \frac{6}{7}, \frac{4}{5}$. *c.* $\frac{3}{5}, \frac{7}{10}, \frac{4}{9}$. *d.* $\frac{1}{2}, \frac{10}{11}, \frac{1}{3}$.
e. $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}$. *f.* $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. *g.* $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}$. *h.* $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$.
i. $\frac{5}{6}, \frac{7}{8}, \frac{8}{9}$. *j.* $\frac{9}{10}, \frac{10}{11}, \frac{11}{12}$. *k.* $\frac{12}{13}, \frac{13}{14}, \frac{14}{15}$. *l.* $\frac{4}{5}, \frac{5}{6}, \frac{6}{8}$.
m. $\frac{10}{11}, \frac{11}{12}, \frac{13}{14}$. *n.* $\frac{14}{15}, \frac{15}{16}, \frac{17}{18}$. *o.* $\frac{19}{20}, \frac{7}{24}, \frac{12}{30}$. *p.* $\frac{8}{9}, \frac{3}{10}, \frac{9}{12}$.
q. $\frac{6}{12}, \frac{4}{16}, \frac{8}{20}$. *r.* $\frac{5}{7}, \frac{1}{8}, \frac{5}{9}$. *s.* $\frac{7}{8}, \frac{35}{72}, \frac{1}{100}$. *t.* $\frac{1}{4}, \frac{1}{10}, \frac{1}{75}$.

GREATEST COMMON DIVISOR

A **common divisor** of two or more numbers is any number greater than 1 that exactly divides each of them.

The **greatest common divisor** of two or more numbers is the greatest number that exactly divides each of them.

Numbers that have no common divisor are said to be **prime** to one another.

When numbers have been resolved into their *prime factors*, their G. C. D. can be obtained by inspection.

1. Find the G. C. D. of 36, 42, and 84.

$$36 = 2 \times 2 \times 3 \times 3$$
$$42 = 2 \times 3 \times 7 \qquad \text{G. C. D. is } 2 \times 3.$$
$$84 = 2 \times 2 \times 3 \times 7$$

The G. C. D. required is 6; for each number is divisible by 2 and by 3, and there is no other prime factor common to all three numbers.

2. Find the G. C. D. of .

a. 8 and 14.	*e.* 12 and 30.	*i.* 16, 64, 256, 1024.
b. 40 and 60.	*f.* 36 and 90.	*j.* 110, 140, and 680.
c. 48 and 144.	*g.* 7, 14, 21.	*k.* 24, 51, 105, 729.
d. 15, 27, 105.	*h.* 32, 48, 128.	*l.* 1326, 3094, 4420.

3. What is the G. C. D. of 1365 and 1995?

3	1365		3	1995	
5	455		5	665	
7	91		7	133	
	13			19	

The prime factors of 1365 are 3, 5, 7, 13.

The prime factors of 1995 are 3, 5, 7, 19.

$$3 \times 5 \times 7 = 105 = \text{G. C. D. of 1365 and 1995}$$

4. Find the G. C. D.:

a. 6, 12, and 30. *d*. 36, 63, and 99. *g*. 25, 35, and 70.
b. 40, 64, and 84. *e*. 12, 18, and 72. *h*. 72 and 168.
c. 15, 25, and 30. *f*. 85 and 95. *i*. 28, 42, and 70.

5. Find the G. C. D. of 304 and 1072. TEACH

304)1072(3
912
160)304(1
160
144)160(1
144
16)144(9
144

We divide 1072 by 304, the remainder is 160; we then divide 304 by 160, the remainder is 16; lastly, we divide 144 by 16, with no remainder.

16)304(19 16)1072(67

We divide the greater of the two given numbers by the smaller, then the divisor by the remainder; next the last divisor by the new remainder, and so on until there is no remainder. The last divisor will be the G. C. D. required.

The G.C.D. of dividend and divisor is also the G. C. D. of divisor and remainder.

If the last divisor is unity, the given numbers have no common measure, that is, they are prime to each other.

6. Find the G. C. D. of: WRITE

a. 1024 and 2240.
b. 11256 and 19899.
c. 1902 and 24409.
d. 5161 and 7755.
e. 1624 and 14500.
f. 87147 and 178871.
g. 8393 and 4609.
h. 1261 and 663.
i. 714 and 1176.
j. 918 and 1347.
k. 219 and 11476.
l. 187 and 255.
m. 194706 and 289913.
n. 1914 and 35786.
o. 2925 and 29484.
p. 21671 and 22111.
q. 27525 and 1725.
r. 82159 and 582.

THE G. C. D. OF SEVERAL NUMBERS

If the G. C. D. of *three* numbers be required, we may first find the G. C. D. of two of the numbers. The G. C. D. of this G. C. D. and the third number will be the G. C. D. required.

1. If we require the G. C. D. of 351, 459, and 1017, we first find the G. C. D. of 351 and 459. This G. C. D. is 27. Next we find the G. C. D. of 27 and 1017. This G. C. D. is 9, which is the G. C. D. required.

2. What is the G. C. D. of:

A 505, 707, and 4343?	B 86, 430, and 473?
C 783, 435, and 555?	D 185, 259, and 407?
E 345, 483, and 609?	F 823, 800, and 672?
G 945, 1560, and 22683?	H 311, 400, and 510?
I 216, 408, and 740?	J 1602, 1603, and 1604?
K 944, 1488, and 2088?	L 492, 744, and 1044?
M 2842, 3654, and 2030?	N 333, 792, and 1665?

A second method of finding the G. C. D. is to find the prime factors as in finding the L. C. M.

3. What is the G. C. D. of 56, 188, 1200?

2	56	188	1200
2	28	94	600
	14	47	300

As 47 is a prime number, the only common factors are 2 and 2. 4 is the G. C. D.

4. Find the G. C. D. of 1058, 2196, 10888, 50024, either by extending the first method or by applying the second.

5. Find the G. C. D. of:

A 556, 672, 452, 212.	B 96, 288, 264, 480.
C 740, 840, 5060, 3080.	D 1825, 2555, 7300.

6. Find the G. C. D. in 2 by the second method.

LEAST COMMON MULTIPLE

A **multiple** of a number is a number exactly divisible by that number.

15, 20, and 45 are multiples of 5.

A **common multiple** of two or more given numbers is any number exactly divisible by each of them.

45 is a common multiple of 3, 5, and 9.

The **least common multiple** of two or more numbers is the least number that is exactly divisible by each of them.

84 is the least common multiple of 3, 7, 14, and 21.

A multiple of a number contains all the prime factors of that number. A multiple of two or more numbers contains all the prime factors of those numbers.

The least number that exactly contains all the prime factors of two or more numbers must be the least common multiple of those numbers.

1. Find the L. C. M. of 12, 20, 30, 54.

2	12	20	30	54
2	6	10	15	27
3	3	5	15	27
5	1	5	5	9
	1	1	1	9

L. C. M. $= 2 \times 2 \times 3 \times 5 \times 9 = 540$

The L. C. M. of two or more numbers must contain all the different prime factors in the numbers; and if a prime factor is repeated in any of them, it must be repeated in the L. C. M. as many times as it is repeated in that number in which it is most often repeated, and no more.

2. Find the L. C. M. of 18, 24, 27, 45.

$18 = 2 \times 3 \times 3$
$24 = 2 \times 2 \times 2 \times 3$
$27 = 3 \times 3 \times 3$
$45 = 3 \times 3 \times 5$

The required L. C. M. is $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 1080$.

LEAST COMMON MULTIPLE

1. Find the least common multiple of 18, 24, 27, 45.

2	18	24	27	45
2	9	12	27	45
2	9	6	27	45
3	9	3	27	45
3	3	1	9	15
	1	1	3	5

The L. C. M.
$= 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$
$= 10 \times 12 \times 9$
$= 1080$

The numbers not exactly divisible by 2 are written in the same line with the quotients, and the division is repeated until only prime numbers are left. The L. C. M. is the product of all the divisors and of the primes in the last line.

2. What is the G. C. D. of the numbers in 1?

3. Find the L. C. M. of:

A	10, 20, 24.	B	2, 4, 6, 8, 10, 12, 16, 18, 20.
C	14, 21, 3, 2, 63.	D	5, 10, 15, 20, 25, 30, 35, 40.
E	24, 16, 18, 20.	F	1, 2, 3, 4, 5, 6, 7, 8, 9.
G	10, 20, 25, 50.	H	5, 9, 15, 18, 36, 135, 162.
I	30, 55.	J	8, 9, 11, 22, 72, 32, 9.
K	6, 9, 30.	L	18, 12, 39, 216, 234.
M	7, 21, 35, 4, 20.	N	8, 18, 15, 20, 70.
O	2, 9, 16, 35, 56, 63.	P	60, 50, 144, 35, 18.
Q	16, 41, 38.	R	27, 54, 81, 14, 63.
S	9, 16.	T	12, 15, 16, 18, 20, 24.
U	112, 200, 72.	V	36, 40, 45, 60, 72, 90.
W	90, 36, 63, 12, 7.	X	115, 184, 230, 460.
Y	3, 5, 7, 9, 11.	Z	140, 168, 210, 280, 420.
&	11, 13, 26, 99, 100.	§	3, 6, 9, 12, 48, 21, 24, 16.

LEAST COMMON DENOMINATOR

The **least common denominator** of two or more fractions is the least denominator to which they all can be reduced. The least common denominator must be the least common multiple of the denominators.

1. Reduce $\frac{5}{6}$, $\frac{8}{9}$, and $\frac{1}{12}$ to their least common denominator.

$$\begin{array}{r|lll} 3 & 6 & 9 & 12 \\ \hline 2 & 2 & 3 & 4 \\ \hline & 1 & 3 & 2 \end{array}$$

$3 \times 2 \times 3 \times 2 = 36$

$6\overline{)36}$ $\quad 5 \times 6 = 30 \quad \frac{5}{6} = \frac{30}{36}$

$9\overline{)36}$ $\quad 8 \times 4 = 32 \quad \frac{8}{9} = \frac{32}{36}$

$12\overline{)36}$ $\quad 1 \times 3 = 3 \quad \frac{1}{12} = \frac{3}{36}$

We find the least common multiple of the denominators, and divide it by the denominator of each fraction. Each quotient multiplied by the numerator of each respective fraction gives the new numerator for the fraction.

2. Reduce $\frac{1}{4}$, $\frac{7}{12}$, $\frac{2}{3}$, and $\frac{9}{16}$ to their least common denominator.

3. Reduce to their least common denominators:

Each fraction should first be reduced to its lowest terms, and mixed numbers should be changed to improper fractions.

a. $\frac{4}{5}, \frac{8}{9}, \frac{3}{15}$. *f.* $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$. *k.* $\frac{2}{5}, \frac{4}{6}, \frac{5}{9}, \frac{7}{10}$. *p.* $\frac{3}{4}, \frac{5}{6}, \frac{3}{5}, \frac{3}{8}, \frac{5}{10}$.

b. $\frac{5}{6}, \frac{4}{9}, \frac{7}{12}$. *g.* $\frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{7}{8}$. *l.* $\frac{67}{120}, \frac{6}{40}, \frac{5}{2}$. *q.* $3\frac{1}{2}, 4\frac{1}{3}, \frac{6}{5}, \frac{9}{15}$.

c. $\frac{41}{50}, 3\frac{6}{20}$. *h.* $\frac{7}{8}, \frac{9}{10}, 7\frac{3}{4}$. *m.* $\frac{5}{12}, \frac{7}{15}, \frac{11}{24}$. *r.* $\frac{3}{7}, \frac{9}{14}, \frac{11}{28}, 5\frac{3}{7}$.

d. $\frac{3}{4}, \frac{4}{5}, \frac{5}{6}$. *i.* $1\frac{1}{2}, \frac{2}{3}, 3\frac{1}{4}$. *n.* $\frac{1}{10}, \frac{1}{3}, \frac{1}{7}, \frac{4}{21}$. *s.* $\frac{2}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{12}, \frac{3}{15}$.

e. $\frac{3}{15}, \frac{4}{24}, \frac{8}{9}$. *j.* $\frac{1}{24}, \frac{1}{36}, \frac{3}{48}$. *o.* $3\frac{1}{8}, 4\frac{4}{12}, 8\frac{6}{18}$. *t.* $\frac{7}{20}, \frac{13}{40}, \frac{29}{80}, \frac{57}{160}$.

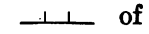
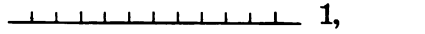
u. $\frac{2}{30}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{9}{20}$. *v.* $\frac{2}{3}, \frac{1}{4}, \frac{3}{5}, \frac{5}{6}, \frac{9}{10}, \frac{7}{120}$. *w.* $\frac{4}{5}, \frac{3}{8}, \frac{4}{6}, \frac{3}{4}, \frac{7}{15}$.

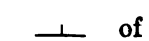
x. $\frac{1}{2}, \frac{2}{8}, \frac{3}{5}, \frac{5}{6}, \frac{7}{8}, \frac{9}{10}, \frac{13}{15}, \frac{7}{16}$. *y.* $\frac{4}{5}, \frac{6}{11}, \frac{29}{20}, \frac{31}{44}, \frac{19}{55}, \frac{3}{4}$.

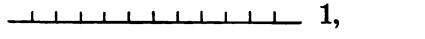
ADDITION OF FRACTIONS

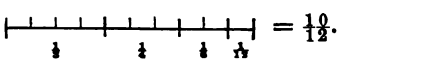
Fractions can be added only when they express fractional parts of the same kind.

1. To add $\frac{1}{3}$ of 1,

$\frac{1}{4}$  of  1,

$\frac{1}{6}$  of 1,

and $\frac{1}{12}$ of  1,

we must reduce $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{12}$ to fractional parts of the same kind. This gives us  $= \frac{10}{12}$.

We reduce the fractions to a common denominator and add. It is usually best to reduce to the least common denominator.

When we have improper fractions and mixed numbers to add together, it is usually better to reduce the improper fractions to mixed numbers, to add the whole numbers, and then to add the fractions when reduced to a least common denominator.

2. Add $\frac{1}{3} + \frac{2}{5} + \frac{4}{15} + \frac{11}{12}$.
3. Add $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{10}$, $\frac{3}{15}$.
4. Add $2\frac{1}{3}$, $3\frac{2}{5}$, $\frac{4}{15}$, 5.
5. Add $3\frac{1}{6}$, $10\frac{1}{9}$, $\frac{105}{12}$.
6. What is the sum of $6\frac{3}{7} + 19\frac{4}{11} + 9\frac{3}{8} + 17\frac{6}{7} + 23\frac{13}{14}$?
7. Add $2\frac{1}{3}$, $3\frac{2}{5}$, $\frac{4}{15}$, $5\frac{11}{12}$.
8. Add $2\frac{1}{2}$, $4\frac{1}{2}$, $5\frac{7}{9}$, $6\frac{2}{3}$.
9. Add $3\frac{1}{3}$, $6\frac{1}{6}$, $9\frac{1}{9}$, $10\frac{3}{12}$.
10. Add $\frac{1}{16}$, $\frac{3}{7}$, $3\frac{1}{4}$, $7\frac{1}{9}$.

11. How many acres are there in 3 lots where the first lot contains $17\frac{3}{5}$ acres, the second $25\frac{2}{6}$ acres, and the third $46\frac{8}{15}$ acres?

12. A man sold $112\frac{6}{7}$ bushels of grain for $250\frac{4}{5}$ dollars; $9\frac{5}{12}$ bushels for $62\frac{3}{8}$ dollars; $225\frac{9}{14}$ bushels for $104\frac{7}{9}$ dollars. How many bushels did he sell? How much money did he receive for the grain?

FRACTIONS

1. Add:

a. $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{3}{8}$, $\frac{8}{11}$.

b. $4\frac{1}{4}$, $1\frac{1}{3}$, $\frac{7}{11}$.

c. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$.

d. $\frac{3}{4}$, $\frac{5}{6}$, $\frac{4}{5}$.

e. $16\frac{3}{11}$, $47\frac{2}{9}$, $21\frac{17}{33}$, $\frac{7}{18}$, $19\frac{1}{2}$.

f. $17\frac{1}{2}$, $43\frac{3}{7}$, $168\frac{4}{9}$, $207\frac{8}{21}$, $506\frac{12}{126}$.

2. A grocer sold $47\frac{3}{7}$ pounds of coffee in one day, $83\frac{3}{5}$ pounds the next day, and $68\frac{5}{9}$ pounds the next day. How many pounds did he sell in the 3 days?

3. I traveled $85\frac{5}{12}$ miles in one day, $78\frac{9}{16}$ in another day, and $125\frac{17}{35}$ in another day. How many miles did I travel in the 3 days?

4. A man bought 3 bolts of cloth, one containing $127\frac{7}{8}$ yards, another $168\frac{9}{16}$ yards, and the third $256\frac{3}{5}$ yards. How many yards did he buy?

5. I bought a table for $\$2\frac{5}{8}$, a sofa for $\$9\frac{5}{8}$, and a bookcase for $\$5\frac{3}{4}$. What was the total cost?

6. A grocer bought 4 loads of potatoes containing respectively $54\frac{3}{4}$, $55\frac{3}{8}$, $51\frac{7}{16}$, and $50\frac{21}{32}$ bushels. How many bushels did he buy?

7. I paid for sugar at one time $\$\frac{7}{12}$, at another time $\$3\frac{3}{8}$, at another $\$7\frac{3}{4}$, and at another time $\$5\frac{1}{6}$. How much did I pay in all?

8. A dealer sold three pieces of silk, the first piece contained $22\frac{5}{6}$ yards, the second $20\frac{7}{8}$ yards, and the third $21\frac{4}{5}$ yards. How many yards were there in the three pieces?

9. I bought 5 tubs of butter; the first tub contained $18\frac{7}{12}$ lb., the second $19\frac{11}{20}$ lb., the third $19\frac{5}{9}$ lb., the fourth $21\frac{11}{15}$ lb., and the fifth $20\frac{13}{18}$ lb. How many pounds of butter were there in the 5 tubs?

SUBTRACTION OF FRACTIONS

What is meant by *subtraction?* Explain *minuend*, *subtrahend*, *remainder*. How many numbers may be added at one time? How many may be subtracted?

One fraction can be subtracted from another only when both express fractional units of the same kind.

1. Find the difference between $\frac{2}{3}$ and $\frac{5}{7}$.

Why can we neither add nor subtract **unlike** fractions?

$$\frac{2}{3} = \frac{14}{21}, \quad \frac{5}{7} = \frac{15}{21}, \quad \frac{5}{7} - \frac{2}{3} = \frac{15}{21} - \frac{14}{21} = \frac{1}{21}$$

First we reduce the fractions to equivalent fractions having the lowest common denominator. Then we subtract the numerator of the subtrahend from the numerator of the minuend, and write the result over the denominator.

2. Find the value of:

a. $\frac{7}{18} - \frac{4}{21}$. *b.* $\frac{19}{20} - \frac{11}{16}$. *c.* $\frac{111}{200} - \frac{1}{10}$. *d.* $\frac{1}{10} - \frac{1}{1000}$.

e. $\frac{17}{24} - \frac{7}{20}$. *f.* $\frac{11}{34} - \frac{1}{10}$. *g.* $\frac{3}{5} - \frac{1}{4}$. *h.* $\frac{12}{15} - \frac{8}{15}$.

i. $\frac{31}{36} - \frac{9}{16}$. *j.* $\frac{18}{37} - \frac{3}{11}$. *k.* $\frac{25}{35} - \frac{17}{35}$. *l.* $\frac{18}{27} - \frac{3}{9}$.

3. From $192\frac{2}{11}$ take $16\frac{15}{16}$.

$\frac{2}{11}$ and $\frac{15}{16}$ reduced to a common denominator become $\frac{32}{176}$ and $\frac{165}{176}$.

$192\frac{2}{11} - 16\frac{15}{16} = 192\frac{32}{176} - 16\frac{165}{176} = 191 + 1\frac{32}{176} - 16\frac{165}{176}$
$= 191\frac{208}{176} - 16\frac{165}{176} = 175\frac{43}{176}$.

Since we cannot subtract $\frac{165}{176}$ from $\frac{32}{176}$, we take 1 from the integral part of the minuend, and considering it as $\frac{176}{176}$ add it to $\frac{32}{176}$. $192\frac{32}{176} = 191\frac{208}{176}$.

4. A boy rode in two days on his bicycle $53\frac{1}{10}$ mi. The first day he rode $29\frac{2}{25}$ mi. How far did he ride the second day?

5. $9\frac{1}{18}$ yd. $- 5\frac{7}{12}$ yd. $= ?$

6. From $29\frac{2}{11}$ take $16\frac{4}{7}$.

$29\frac{2}{11} - 16\frac{4}{7} = 29\frac{14}{77} - 16\frac{44}{77} = 28 + 1\frac{14}{77} - 16\frac{44}{77} = 28\frac{91}{77} - 16\frac{44}{77} = 12\frac{47}{77}$.

7. From $117\frac{3}{18}$ take $67\frac{40}{45}$.

8. Find the value of:

a. $1\frac{21}{40} - \frac{36}{47}$. *b.* $2\frac{63}{74} - 1\frac{58}{63}$. *c.* $46\frac{5}{16} - 27\frac{4}{19}$. *d.* $19\frac{3}{8} - 14\frac{7}{11}$.

9. Subtract:

a. $31\frac{1}{4}$	*b.* $42\frac{4}{7}$	*c.* $16\frac{6}{16}$	*d.* $15\frac{2}{3}$	*e.* $13\frac{3}{7}$
$6\frac{1}{4}$	$18\frac{3}{8}$	$5\frac{2}{9}$	$\frac{3}{4}$	$9\frac{1}{7}$
f. $62\frac{1}{2}$	*g.* 38	*h.* 14	*i.* 105	*j.* $25\frac{4}{5}$
$\frac{7}{8}$	$\frac{11}{15}$	$\frac{6}{13}$	$62\frac{1}{2}$	$6\frac{2}{5}$
k. $91\frac{1}{8}$	*l.* $72\frac{3}{5}$	*m.* 75	*n.* $57\frac{1}{9}$	*o.* $18\frac{3}{4}$
$\frac{9}{10}$	$24\frac{7}{9}$	$18\frac{3}{4}$	$\frac{4}{5}$	$12\frac{1}{2}$
p. $8\frac{6}{17}$	*q.* 21	*r.* 30	*s.* $144\frac{4}{19}$	*t.* $37\frac{1}{2}$
$3\frac{5}{7}$	$4\frac{13}{14}$	$10\frac{3}{7}$	$98\frac{7}{19}$	$6\frac{1}{4}$

REVIEW

10. What is the area of a parallelogram whose base is 15.23 ft. long, and whose altitude is 19.64 ft.?

11. Multiply 1373.78 by 29, 36, 58, and 47.

12. Write in words 10056932812647829032.

13. What is the area of a rectangle 16 mi. by 32 mi.?

14. Change to decimals:

a. $\frac{2}{7}$ *b.* $\frac{3}{15}$ *c.* $\frac{9}{12}$ *d.* $\frac{1}{99}$ *e.* $\frac{49}{500}$

15. In a town the tax rate for schools was .92 mills per dollar. What was the amount of the tax on a total valuation of $9,875,632.58?

16. The distance of the sun from the earth is 93,000,000 miles. Find its ratio to the moon's distance, 240,000 miles.

ADDITION OF MIXED NUMBERS

1. Add $2\frac{1}{2}$, $3\frac{2}{3}$, $4\frac{5}{6}$, $8\frac{3}{4}$, $10\frac{2}{3}$, $7\frac{5}{8}$.

$2 + 3 + 4 + 8 + 10 + 7 = 34$, the sum of the integers.

2	2	3	6	4	8
2	1	3	3	2	4
3	1	3	3	1	2
	1	1	1	1	2

$2 \times 2 \times 3 \times 2 = 24$, the least common multiple of the denominators of the fractions.

$$\tfrac{1}{2} = \tfrac{12}{24} \quad \tfrac{2}{3} = \tfrac{16}{24} \quad \tfrac{5}{6} = \tfrac{20}{24} \quad \tfrac{3}{4} = \tfrac{18}{24} \quad \tfrac{2}{3} = \tfrac{16}{24} \quad \tfrac{5}{8} = \tfrac{15}{24}$$

$$\tfrac{12}{24} + \tfrac{16}{24} + \tfrac{20}{24} + \tfrac{18}{24} + \tfrac{16}{24} + \tfrac{15}{24} = \tfrac{97}{24} = 4\tfrac{1}{24}$$

$$34 + 4\tfrac{1}{24} = 38\tfrac{1}{24}$$

Another way is to reduce all the mixed numbers to improper fractions with a common denominator.

$$2\tfrac{1}{2} = \tfrac{5}{2} \quad 3\tfrac{2}{3} = \tfrac{11}{3} \quad 4\tfrac{5}{6} = \tfrac{29}{6} \quad 8\tfrac{3}{4} = \tfrac{35}{4} \quad 10\tfrac{2}{3} = \tfrac{32}{3} \quad 7\tfrac{5}{8} = \tfrac{61}{8}$$

$$\tfrac{5}{2} = \tfrac{60}{24} \quad \tfrac{11}{3} = \tfrac{88}{24} \quad \tfrac{29}{6} = \tfrac{116}{24} \quad \tfrac{35}{4} = \tfrac{210}{24} \quad \tfrac{32}{3} = \tfrac{256}{24} \quad \tfrac{61}{8} = \tfrac{183}{24}$$

$$\tfrac{60}{24} + \tfrac{88}{24} + \tfrac{116}{24} + \tfrac{210}{24} + \tfrac{256}{24} + \tfrac{183}{24} = \tfrac{913}{24} = 38\tfrac{1}{24}$$

2. Add $1\frac{3}{4}$, $2\frac{5}{6}$, $4\frac{1}{3}$, $5\frac{3}{8}$, $6\frac{2}{3}$, $7\frac{1}{2}$.

3. Add $1\frac{3}{8}$, $2\frac{3}{4}$, $3\frac{2}{3}$, $4\frac{1}{2}$, $5\frac{1}{6}$, $8\frac{1}{4}$.

4. Add $\$3\frac{3}{4}$, $\$5\frac{1}{6}$, $\$8\frac{3}{8}$, $\$9\frac{1}{8}$, $\$10\frac{1}{2}$.

5. A boy was sent by his father to collect three bills. His father gave him $\$2\frac{3}{4}$ in change; and he collected from one man $\$2\frac{1}{2}$, from another $\$6\frac{5}{10}$, and from another $\$10\frac{3}{5}$. How much money in all did he then have to hand to his father?

6. Prove the answer in 5 by reducing the money to decimal amounts, and adding.

FRACTION PROBLEMS

1. I bought cloth at $\$1\frac{7}{8}$ a yard, and sold it at $\$2\frac{1}{4}$ a yard. What did I gain on a yard?

2. From a barrel of vinegar containing $31\frac{1}{2}$ gallons, $12\frac{5}{8}$ gallons were drawn. How many gallons were left?

3. I purchased $10\frac{3}{8}$ tons of coal at one time, and $24\frac{5}{6}$ tons at another; after using $16\frac{7}{12}$ tons, how much remained?

4. A boy having $\$\frac{3}{4}$, gave $\$\frac{1}{5}$ for oranges, and $\$\frac{1}{4}$ for a flag. How much money had he left?

5. A man spent $\$15\frac{7}{16}$ for provisions, and $\$12\frac{7}{8}$ for clothing. How much money had he left from $\$50\frac{1}{2}$?

6. A merchant bought two pieces of velvet, one containing $54\frac{9}{10}$ yards, and the other $56\frac{11}{12}$ yards. He sold $43\frac{13}{15}$ yards at one time, and $34\frac{4}{5}$ yards at another. How many yards had he left?

7. A grocer bought two tubs of butter, one tub containing $27\frac{4}{5}$ pounds, the other $32\frac{1}{8}$ pounds. After he sold $40\frac{17}{18}$ pounds, how many pounds had he left?

8. It is $95\frac{3}{4}$ feet from one house-lot on a city street to the opposite, and the sidewalks on the street are $12\frac{1}{2}$ feet wide. How wide is the street between the sidewalks?

9. The Capitol at Washington is 324 feet wide and $752\frac{1}{3}$ feet long. What is the difference between its length and its width?

10. From a piece of cloth containing $34\frac{3}{8}$ yards, there were sold $12\frac{2}{3}$ yards. How many yards were left?

11. There was a total eclipse of the sun on May 28th, 1900. The path of the eclipse crossed Alabama. The region in temporary darkness was $53\frac{7}{8}$ miles wide. If the eastern boundary of Alabama is $331\frac{1}{2}$ miles long, how many miles of the boundary were not in the path of the eclipse?

MULTIPLICATION OF FRACTIONS

Multiplying the numerator has what effect on the fraction? Dividing the denominator has what effect? Why?

If the numerator of the fraction $\frac{15}{16}$ be divided by 5, how will the fraction be affected?

If the numerator be multiplied by 4, how will the fraction be affected?

What difference will be made in the value of the fraction if the denominator is multiplied by 2? If the denominator is divided by 2?

1. Multiply $\frac{8}{17}$ by 4.

This means repeat $\frac{3}{17}$ four times.

$$\tfrac{3}{17}+\tfrac{3}{17}+\tfrac{3}{17}+\tfrac{3}{17}=\tfrac{12}{17}. \quad \tfrac{3}{17}\times 4=\tfrac{12}{17}.$$

2. Multiply $\frac{11}{36}$ by 18.

$$\frac{11}{36}\times 18=\frac{11}{\cancel{36}_{2}}\times\overset{1}{\cancel{18}}=\frac{11}{2}=5\tfrac{1}{2}$$

First we cancel the factors common to denominator and multiplier. Then we multiply. The resulting fraction we reduce to a mixed number.

3. Multiply $\frac{7}{9}\times 6$.

$$\frac{7}{\cancel{9}_{3}}\times\overset{2}{\cancel{6}}=\frac{14}{3}=4\tfrac{2}{3}$$

4. Multiply $1\frac{3}{4}\times 16$.

$$1\tfrac{3}{4}=\frac{7}{4} \qquad \frac{7}{\cancel{4}_{1}}\times\overset{4}{\cancel{16}}=\frac{28}{1}=28$$

5. Multiply:

a. $\frac{13}{24}\times 8$. *b.* $\frac{5}{8}\times 4$. *c.* $\frac{3}{16}\times 16$. *d.* $\frac{47}{49}\times 7$.

e. $\frac{175}{45}\times 9$. *f.* $\frac{127}{15}\times 5$. *g.* $\frac{369}{147}\times 49$. *h.* $\frac{21}{30}\times 15$.

i. $\frac{17}{40}\times 8$. *j.* $\frac{161}{180}\times 45$. *k.* $12\frac{2}{3}\times 8$. *l.* $\frac{28}{7}\times 14$.

6. Multiply 24 by $\frac{3}{8}$.

$$24 \times \frac{3}{8} = 9$$

7. Multiply: *a.* 360 by $\frac{2}{3}$.
b. 672 by $\frac{5}{6}$.
c. 580 by $\frac{3}{4}$.
d. 24 by $\frac{7}{8}$.

8. Find the products:

a. $136 \times \frac{1}{3}$.	*b.* $672 \times \frac{5}{6}$.	*c.* $360 \times \frac{2}{3}$.	*d.* $45 \times \frac{7}{9}$.
e. $432 \times \frac{1}{4}$.	*f.* $710 \times \frac{7}{8}$.	*g.* $580 \times \frac{3}{4}$.	*h.* $32 \times 2\frac{3}{8}$.
i. $635 \times \frac{1}{5}$.	*j.* $660 \times \frac{3}{2\,6}$.	*k.* $765 \times \frac{11}{12}$.	*l.* $50 \times \frac{11}{14}$.
m. $840 \times \frac{49}{80}$.	*n.* $24 \times \frac{7}{8}$.	*o.* $975 \times \frac{145}{463}$.	*p.* $28 \times 3\frac{2}{3}$.
q. $42 \times \frac{11}{12}$.	*r.* $64 \times \frac{15}{36}$.	*s.* $105 \times \frac{5}{7}$.	*t.* $25 \times \frac{3}{4}$.

9. $\frac{3}{4} \times 8 = \frac{3}{4}$ of $8 = 8 \times \frac{3}{4}$.

To multiply a whole number by a fraction is the same as to multiply a fraction by a whole number.

We may take three fourths of eight things or eight three-fourths of something, with the same result.

10. Discuss in this way:

a. $9\not{\text{¢}} \times \frac{2}{3}$ or $\frac{2}{3}\not{\text{¢}} \times 9$. *b.* 27 bu. $\times \frac{1}{9}$ or $\frac{1}{9}$ bu. $\times$ 27.

c. 16 lb. $\times \frac{1}{8}$ or $\frac{1}{8}$ lb. $\times$ 16. Would it make any difference in the total amount of tea used, if we put $\frac{1}{8}$ of 16 lb. in one package or $\frac{1}{8}$ lb. in each of 16 packages?

d. 10 yd. $\times \frac{2}{5}$ or $\frac{2}{5}$ yd. $\times$ 10. *e.* 60 min. $\times \frac{1}{4}$ or $\frac{1}{4}$ min. $\times$ 60.

11. Multiply 15 minutes by $\frac{4}{5}$.

This means, take four fifths of fifteen minutes.

$15 \div 5 = 3$. One fifth of fifteen minutes is three minutes.

3 min. $\times$ 4 = 12 min. Three minutes multiplied by four is twelve minutes.

12. Discuss multiplying:

a. 200 qt. by $\frac{3}{10}$. *b.* 30 days by $\frac{5}{6}$. *c.* 365 days by $\frac{1}{5}$.

FRACTIONS OF FRACTIONS

1. Multiply $\frac{3}{5}$ by $\frac{2}{7}$.

This means that the seventh part of $\frac{3}{5}$ is to be taken twice. The seventh part of $\frac{3}{5}$ is $\frac{3}{35}$. Twice $\frac{3}{35}$ is $\frac{6}{35}$.

$$\frac{3}{5} \times \frac{2}{7} = \frac{6}{35}$$

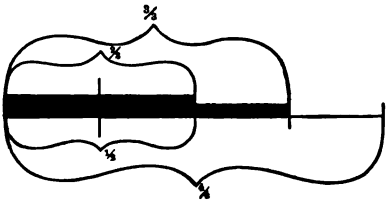
The product of the two fractions is equivalent to the product of the numerators divided by the product of the denominators.

$$3 \times 2 = 6 \qquad 5 \times 7 = 35$$

$\frac{3}{5} \times \frac{2}{7}$ may be written as $\frac{3}{5}$ of $\frac{2}{7}$, or $\frac{2}{7}$ of $\frac{3}{5}$.

2. Multiply $\frac{2}{3}$ by $\frac{3}{4}$. $\quad \frac{\overset{1}{\cancel{2}}}{\cancel{3}} \times \frac{\overset{1}{\cancel{3}}}{\cancel{4}_{2}} = \frac{1}{1} \times \frac{1}{2} = \frac{1}{2}$

This means, take two thirds of three fourths.



In canceling the common factors, it is best to write 1 when it is a quotient. This helps us not to overlook any of the numbers or factors.

WRITE

3. Multiply $\frac{3}{5}$ by $\frac{4}{7}$ by $\frac{6}{11}$.

This means, take $\frac{6}{11}$ of $\frac{4}{7}$ of $\frac{3}{5}$, or $\frac{3}{5} \times \frac{4}{7} \times \frac{6}{11} = ?$

$$\frac{3}{5} \times \frac{4}{7} \times \frac{6}{11} = \frac{3 \times 4 \times 6}{5 \times 7 \times 11} = \frac{72}{385}$$

4. Multiply:

a. $\frac{4}{5} \times \frac{10}{16}$. *b.* $\frac{3}{5} \times \frac{5}{7}$. *c.* $\frac{4}{9} \times \frac{18}{27}$. *d.* $\frac{8}{11} \times \frac{55}{56}$.

e. $\frac{7}{9} \times \frac{63}{77}$. *f.* $\frac{13}{14} \times \frac{28}{120}$. *g.* $\frac{11}{17} \times \frac{68}{120}$. *h.* $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}$.

i. $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$. *j.* $\frac{3}{7} \times \frac{7}{9} \times \frac{8}{15}$. *k.* $\frac{5}{6} \times \frac{4}{7} \times \frac{6}{11}$. *l.* $\frac{2}{3} \times \frac{4}{5} \times \frac{7}{10}$.

m. $\frac{5}{12} \times \frac{9}{20} \times \frac{6}{27}$. *n.* $\frac{7}{11} \times \frac{22}{23} \times \frac{46}{49}$. *o.* $\frac{14}{27} \times \frac{9}{28} \times \frac{6}{13} \times \frac{26}{30}$.

5. Multiply together $\frac{4}{9}$, $\frac{3}{11}$, $6\frac{2}{7}$, $9\frac{3}{5}$, $2\frac{1}{2}$, 63. These numbers may be written:

$$\frac{4}{9}\times\frac{3}{11}\times\frac{44}{7}\times\frac{48}{5}\times\frac{5}{2}\times\frac{63}{1}=\frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{9}}}\times\frac{3}{\underset{1}{\cancel{11}}}\times\frac{\overset{4}{\cancel{44}}}{\underset{1}{\cancel{7}}}\times\frac{48}{\underset{1}{\cancel{5}}}\times\frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{2}}}\times\frac{\overset{1}{\cancel{7}}\cancel{63}}{1}$$

$$=\frac{2\times3\times4\times1\times48}{1\times1\times1\times1\times1}=1152$$

6. Find the value of:

a. $\frac{1}{2}\times\frac{3}{7}\times\frac{9}{11}\times\frac{4}{81}\times9$. *b.* $3\times7\frac{1}{3}\times\frac{11}{15}\times3\frac{8}{11}$.

7. What will $12\frac{3}{4}$ pounds of spice cost at $\$\frac{8}{7}$ a pound?

8. What will $3\frac{1}{2}$ bushels of onions cost at $\$\frac{7}{8}$ a bushel?

9. At 75¢ a bushel, what will $\frac{11}{15}$ bu. of corn cost?

10. If a man earns $\$1\frac{7}{8}$ in one day, how much can he earn in 24 days?

11. $\frac{1}{17\frac{1}{9}}\times18\frac{7}{11}\times9\frac{3}{5}\times\frac{1}{2}\times\frac{3}{4}\times7\times\frac{3}{5}\times\frac{11}{14}\times25=?$

12. At $\$16$ per M, what will $\frac{11}{12}$ of a thousand feet of lumber cost?

13. If 1 yd. of cloth cost $\$1\frac{1}{4}$, what will $6\frac{1}{8}$ yd. cost?

14. What will $3\frac{3}{8}$ bu. of peaches cost at $\$2\frac{1}{2}$ per bu.?

15. If a quantity of lumber is valued at $\$75\frac{8}{15}$, what will be the value of $\frac{5}{12}$ of it?

16. If A pays $\$56$ as railroad fare for a certain distance, how much should he pay for $\frac{9}{14}$ of that distance?

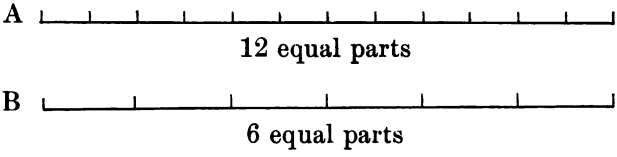
17. If a roll of wire is worth $\$\frac{5}{6}$, what is $\frac{2}{3}$ of a roll worth?

18. I owned $\frac{7}{15}$ of a vessel and sold $\frac{9}{20}$ of my share. What part of the vessel did I sell?

19. A tailor used $\frac{9}{10}$ of a yard of silesia and $\frac{5}{12}$ as much lasting. How much lasting did he use?

20. $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{8}{9}$ of $\frac{6}{25}$ of $\$50=?$

DIVISION OF FRACTIONS



In A each part is $\frac{1}{2}$ the size of each part of B.

In B each part is 2 times the size of each part of A.

$\frac{1}{12}$ is $\frac{1}{2}$ of $\frac{1}{6}$ $\qquad$ $\frac{1}{6} = 2 \times \frac{1}{12}$ $\qquad$ $\frac{1}{12} = \frac{1}{2} \times \frac{1}{6}$

$\frac{1}{6} \div 2 = \frac{1}{12}$

We may divide a fraction by a whole number by multiplying the denominator by that number.

If $\frac{1}{6} \div 2 = \frac{1}{12}$, then $\frac{2}{6} \div 2 = \frac{2}{12}$.

But $\frac{2}{12} = \frac{1}{6}$.

We may divide a fraction by a whole number by dividing the numerator by that number.

$\frac{3}{12} \div 3 = \frac{3}{36}$ or $\frac{1}{12}$. But $\frac{3}{36} = \frac{1}{12}$.

Dividing the numerator of a fraction by a number or multiplying the denominator by it divides the fraction.

1. Divide:

a. $\frac{1}{8}$ by 3. *b.* $\frac{3}{25}$ by 5. *c.* $\frac{8}{21}$ by 4. *d.* $\frac{15}{100}$ by 5.

e. $\frac{7}{8}$ by 4. *f.* $\frac{9}{10}$ by 3. *g.* $\frac{18}{20}$ by 10. *h.* $\frac{75}{100}$ by 25.

2. Six boys had to run a relay race of two miles. What part of a mile did each boy run?

3. Five boys had a quarter of a dollar. What part of a dollar did each boy have as his share?

4. Three boys had a half dollar. They spent each his share. What part of a dollar did each spend?

5. How much is one half of a half dollar? one fifth?

MULTIPLICATION BY MIXED NUMBERS

1. What is the product of $3\frac{2}{3} \times 4\frac{7}{8} \times 1\frac{5}{22}$?

$$3\tfrac{2}{3} = \tfrac{11}{3} \qquad 4\tfrac{7}{8} = \tfrac{39}{8} \qquad \frac{\overset{1}{\cancel{11}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{13}{\cancel{39}}}{8} \times \frac{15}{\underset{2}{\cancel{22}}} = \frac{13 \times 15}{8 \times 2} = \tfrac{195}{16} = 12\tfrac{3}{16}$$

We reduce the mixed numbers to improper fractions, cancel the factors found in both the numerators and the denominators, multiply together the remaining values of the numerators for the numerator of the product and of the denominators for the denominator of the product, and reduce the result to lowest terms.

2. Find the products:

a. $5\frac{6}{7} \times 3\frac{1}{2}$.

b. $67 \times 9\frac{1}{2}$.

c. $3\frac{1}{3} \times \frac{1}{5} \times 68$.

d. $\frac{1}{2} \times 75 \times \frac{2}{3} \times 28$.

e. $6\frac{1}{5} \times 2\frac{7}{9} \times \frac{1}{4} \times 12$.

f. $12\frac{3}{4} \times 11\frac{5}{9}$.

g. $8\frac{5}{11} \times 6\frac{2}{7}$.

h. $12\frac{2}{3} \times 9$.

i. $4\frac{7}{8} \times 9\frac{1}{4}$.

j. $15\frac{3}{4} \times 20\frac{8}{9}$.

k. $1475 \times \frac{7}{9} \times \frac{21}{100}$.

l. $\frac{800}{12} \times \frac{3}{8} \times \frac{1000}{9}$.

m. $13\frac{2}{3} \times 17\frac{7}{9}$.

n. $7\frac{1}{8} \times 8\frac{3}{7}$.

o. $2\frac{1}{3} \times \frac{2}{3} \times \frac{3}{4} \times 85$.

p. $6\frac{3}{4} \times 2\frac{14}{27}$.

q. $11\frac{2}{7} \times 8\frac{4}{5}$.

r. $108 \times 12\frac{4}{9}$.

s. $30\frac{5}{6} \times 44\frac{4}{20}$.

t. $\frac{6}{7} \times 2\frac{1}{2} \times \frac{3}{5} \times 61$.

u. $\frac{2}{3} \times 16\frac{1}{8} \times \frac{6}{7} \times 9\frac{1}{3}$.

v. $\frac{23}{47} \times 65\frac{1}{3} \times 46\frac{1}{9}$.

w. $\frac{7}{8} \times \frac{9}{10} \times 20 \times 25\frac{1}{2}$.

x. $\frac{7}{9} \times 3\frac{4}{81} \times 4\frac{1}{2} \times \frac{2}{5} \times 6\frac{1}{7} \times 5$.

y. $\frac{7}{8} \times \frac{5}{6} \times \frac{7}{16}$.

z. $14 \times 15\frac{1}{16} \times 3\frac{5}{9}$.

aa. $\frac{9}{10} \times 8\frac{3}{4} \times \frac{9}{11} \times \frac{11}{12}$.

bb. $\frac{6}{7} \times \frac{11}{8} \times \frac{6}{33} \times 21 \times \frac{3}{5}$.

cc. $\frac{1}{4} \times 8 \times \frac{2}{7} \times 19$.

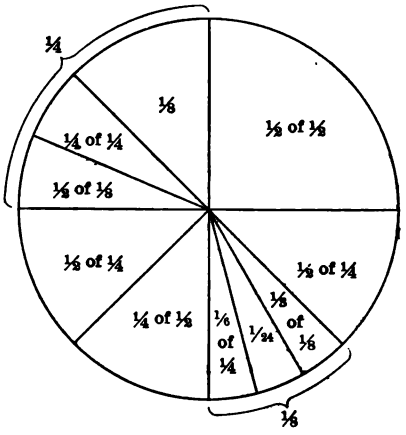
dd. $\frac{9}{10} \times 7 \times \frac{11}{15} \times 87\frac{3}{11}$.

ee. $3\frac{2}{3} \times 4\frac{7}{8} \times 15$.

ff. $\$7 \times \frac{3}{5} \times 2\frac{1}{4} \times \frac{1}{10} \times 8$.

FRACTIONS BY FRACTIONS

MULTIPLICATION



Multiplying a fraction by a fraction is always exactly the same as taking a fraction of a fraction.

$\frac{1}{2} \times \frac{1}{4} = \frac{1}{2}$ of $\frac{1}{4}$

DIVISION

Dividing a fraction by a fraction is always exactly the opposite of taking a fraction of a fraction. Division as a process is the opposite of multiplication.

1. Divide $\frac{1}{2}$ by 2. $\frac{1}{2} \div 2 = \frac{1}{4}$ $\frac{1}{4}$ is $\frac{1}{2}$ of $\frac{1}{2}$
2. Divide $\frac{1}{2}$ by $\frac{1}{2}$. $1 \div 1 = 1$ $2 \div 2 = 1$ $\frac{1}{2} \div \frac{1}{2} = 1$

$2 = \frac{1}{2} \times 4$ Since $\frac{1}{2} \div 2 = \frac{1}{4}$, then $\frac{1}{2} \div \frac{1}{2} = 1$, since $\frac{1}{2}$ is now divided by a number only $\frac{1}{4}$ of 2.

The smaller the number we divide by, the larger the quotient. A simple fraction is even smaller than 1. Dividing a number by a simple fraction gives a quotient larger than the dividend. Dividing a fraction by a simple fraction gives a quotient larger than the dividend-fraction.

3. How many times does $\frac{1}{4}$ contain $\frac{1}{4}$? $\frac{1}{8}$?
4. $\frac{1}{3})\frac{2}{3} = 2$, since $\frac{2}{3}$ contains 2 $\frac{1}{3}$'s.

DIVISION OF FRACTIONS

1. How many times is $\frac{1}{3}$ contained in $\frac{3}{4}$? or, $\frac{3}{4} \div \frac{1}{3} = ?$

If we were to divide $\frac{3}{4}$ by 1, our answer would be $\frac{3}{4}$. But our divisor is only $\frac{1}{3}$ of 1. Since $1 = 3 \times \frac{1}{3}$, our quotient is $3 \times \frac{3}{4} \div 1$.

$$\tfrac{3}{4} \div \tfrac{1}{3} = \tfrac{3}{4} \times \tfrac{3}{1} = \tfrac{9}{4}$$

2. How many times is $\frac{2}{3}$ contained in $\frac{3}{4}$?

$$\tfrac{3}{4} \div \tfrac{1}{3} = \tfrac{9}{4} \qquad \tfrac{2}{3} = 2 \times \tfrac{1}{3} \qquad \tfrac{3}{4} \div \tfrac{2}{3} = \tfrac{9}{8}$$

3. Divide $\frac{5}{7}$ by $\frac{3}{8}$.

$$\frac{5}{7} \div 3 = \frac{5}{7 \times 3} = \frac{5}{21} \qquad \frac{5}{21} \div \frac{1}{8} = \frac{5}{21} \times \frac{8}{1} = \frac{40}{21}$$

$$\frac{5}{7} \div \frac{3}{8} = \frac{5 \times 8}{7 \times 3} = \frac{40}{21} = 1\tfrac{19}{21}$$

When we divide a fraction by a fraction, we multiply the numerator of the dividend by the denominator of the divisor for the numerator of the quotient, and the denominator of the dividend by the numerator of the divisor for the denominator of the quotient.

To divide a fraction by a fraction invert the divisor and multiply.

4. Divide $\frac{7}{12}$ by $\frac{4}{9}$.

$$\frac{7}{12} \div \frac{4}{9} = \frac{7 \times 9}{12 \times 4} = \frac{\overset{21}{\cancel{63}}}{\underset{16}{\cancel{48}}} = \frac{21}{16} = 1\tfrac{5}{16}$$

5. Divide:

a. $\frac{4}{7}$ by $\frac{9}{10}$.	*e.* $\frac{1}{7}$ by $\frac{3}{8}$.	*i.* $\frac{8}{15}$ by $\frac{12}{20}$.	*m.* $\frac{2}{3}$ by $\frac{7}{8}$.
b. $\frac{1}{5}$ by $\frac{1}{6}$.	*f.* $\frac{3}{4}$ by $\frac{1}{3}$.	*j.* $\frac{7}{9}$ by $\frac{6}{7}$.	*n.* $\frac{5}{9}$ by $\frac{1}{6}$.
c. $\frac{7}{8}$ by $\frac{9}{10}$.	*g.* $\frac{2}{3}$ by $\frac{27}{28}$.	*k.* $\frac{9}{13}$ by $\frac{11}{16}$.	*o.* $\frac{8}{11}$ by $\frac{7}{22}$.
d. $\frac{5}{6}$ by $\frac{6}{7}$.	*h.* $\frac{1}{9}$ by $\frac{3}{10}$.	*l.* $\frac{9}{20}$ by $\frac{12}{13}$.	*p.* $\frac{9}{10}$ by $\frac{2}{7}$.

DIVISION OF FRACTIONS

In multiplying a fraction by a fraction we multiply the numerators together and the denominators together.

We divide a fraction by a fraction by inverting the terms of the divisor and proceeding as in multiplication of fractions.

1. Divide $\frac{5}{8}$ by $\frac{3}{9}$.

$\frac{5}{8} \div \frac{3}{9}$ means

1st. In $\frac{5}{8}$ are how many ninths? 9 times as many eighths as one unit contains ninths; that is, $\frac{5}{8} \times \frac{9}{1} = \frac{45}{8}$.

2d. In $\frac{45}{8}$ are how many combinations of 3-ninths? Evidently $\frac{1}{3}$ as many as there are single or 1-ninths; that is, $\frac{45}{8} \div 3 = \frac{15}{8} = 1\frac{7}{8}$.

2. What do these expressions equal?

a. $\frac{6}{7} \div \frac{5}{8}$. *b.* $\frac{2}{3} \div \frac{5}{6}$. *c.* $\frac{1}{9} \div \frac{7}{25}$. *d.* $\frac{3}{8} \div \frac{4}{9}$.

3. Divide:

a. \$$\frac{3}{4}$ by $\frac{1}{8}$ *b.* \$$\frac{3}{8}$ by $\frac{1}{20}$. *c.* $\frac{1}{10}$ gal. by $\frac{3}{4}$. *d.* $\frac{1}{25}$ T. by $\frac{3}{5}$.

4. If $\frac{1}{6}$ yard of velvet cost \$$\frac{3}{4}$, how much will 1 yard cost?

$$\$\tfrac{3}{4} \div \tfrac{1}{6} = \$\tfrac{3}{4} \times \tfrac{6}{1} = \$\tfrac{9}{2} = \$4\tfrac{1}{2} = \$4.50.$$

5. Discuss each step in **4**.

DIVISION OF MIXED NUMBERS

In order to divide mixed numbers we first reduce the mixed numbers to improper fractions.

6. Divide $2\frac{1}{2}$ by $1\frac{3}{4}$.

$$2\tfrac{1}{2} \div 1\tfrac{3}{4} = \tfrac{5}{2} \div \tfrac{7}{4} = \tfrac{5}{2} \times \tfrac{4}{7} = \tfrac{10}{7} = 1\tfrac{3}{7}$$

7. Divide:

a. $8\frac{1}{4}$ by $6\frac{3}{5}$. *b.* $3\frac{1}{3}$ by $2\frac{10}{21}$. *c.* $9\frac{2}{5}$ by $1\frac{3}{9}$. *d.* $7\frac{1}{2}$ by $3\frac{1}{3}$.

e. $4\frac{2}{3}$ by $1\frac{1}{7}$. *f.* $1\frac{9}{10}$ by $3\frac{1}{9}$. *g.* $1\frac{4}{5}$ by $5\frac{1}{5}$. *h.* $2\frac{1}{2}$ by $1\frac{1}{4}$.

REVIEW OF FRACTIONS

1. Divide:

a. 6 by $1\frac{2}{4}$.	*f.* $\frac{15}{88}$ by 20.	*k.* 212 by $\frac{15}{16}$.	*p.* $19\frac{1}{2}$ by $1\frac{7}{8}$.
b. $\frac{18}{19}$ by 6.	*g.* $\frac{405}{19}$ by 15.	*l.* 63 by $\frac{18}{27}$.	*q.* $73\frac{1}{2}$ by $9\frac{4}{5}$.
c. $\frac{15}{11}$ by 6.	*h.* $\frac{126}{651}$ by 27.	*m.* 420 by $\frac{9}{11}$.	*r.* $54\frac{38}{48}$ by $25\frac{5}{6}$.
d. $\frac{42}{91}$ by 12.	*i.* $\frac{450}{581}$ by 75.	*n.* $1\frac{3}{4}$ by 5.	*s.* $15\frac{1}{4}$ by $30\frac{1}{2}$.
e. $\frac{18}{37}$ by 9.	*j.* 14 by $\frac{7}{8}$.	*o.* $8\frac{1}{6}$ by $\frac{2}{3}$.	*t.* $\frac{3}{1000}$ by $1\frac{1}{2}$.

2. If 1 yard of percale cost $\$\frac{3}{8}$, how many yards can be bought for $\$\frac{9}{16}$?

3. What number multiplied by $5\frac{1}{3}$ will give 146 for the product?

4. The dividend is $520\frac{1}{5}$ and the quotient is $36\frac{9}{10}$. What is the divisor?

5. What number is that which multiplied by $\frac{5}{8}$ of $\frac{3}{7}$ of $15\frac{1}{8}$ will produce $\frac{5}{8}$?

6. A man spent $\$165\frac{3}{10}$ in $8\frac{1}{2}$ weeks. How much did he spend in 1 week?

7. At $\$9\frac{3}{8}$ a cord, how many cords of wood can be bought for $\$138\frac{3}{4}$?

8. I spent $\$3\frac{4}{5}$ in 8 days. How much did I spend in 1 day at that rate?

9. If 8 yards of serge cost $\$7\frac{21}{25}$, what is the price a yard?

10. If $\frac{4}{5}$ of a tub of butter sells for $\$10\frac{2}{3}$, what should a tub sell for?

11. What will 493 lb. of tea cost at $\$\frac{5}{8}$ per lb.?

12. What will 1286 bu. of apples cost at $\$\frac{21}{25}$ per bu.?

13. How much money will 227 yd. of cloth cost at $\$\frac{3}{4}$ per yd.?

14. What will be the cost of 8750 sq. ft. of land at $\$\frac{7}{10}$ per sq. ft.?

RATIOS OF NUMBERS

1. What part of 4 is 1?

1 is one fourth of 4, because 4 times 1 is 4.

The part one number is of another is its ratio to that number.

2. What part:

a. of 6 is 1? *b.* of 9 is 1? *c.* of 16 is 1? *d.* of 245 is 1?

3. What part of 8 is 3?

Since $1 = \frac{1}{8}$ of 8, 3 must equal $\frac{3}{8}$ of 8.

4. What part:

a. of 9 is 4? *b.* of 3 is 2? *c.* of 6 is 5? *d.* of 11 is 7?
e. of 8 is 5? *f.* of 13 is 4? *g.* of 18 is 15? *h.* of 9 is 6?

5. What part of 7 is 9?

Since $1 = \frac{1}{7}$ of 7, 9 must equal $\frac{9}{7}$ of 7, or $1\frac{2}{7}$ of 7.

6. What part:

a. of 9 is 13? *b.* of 47 is 63? *c.* of 10 is 87?
d. of 39 is 19? *e.* of 19 is 39? *f.* of 413 is 527?

7. What part:

a. of 9 ft. is 1 ft.? *b.* of 4 cwt. is 3 cwt.?
c. of 4 m. is 3 m.? *d.* of 10 da. is 7 da.?
e. of 4 lb. is 3 lb.? *f.* of 8 yr. is 5 yr.?

8. What part of $\frac{4}{7}$ is $\frac{3}{7}$?

$\frac{3}{7}$ is the same part of $\frac{4}{7}$ that 3 is of 4, which is $\frac{3}{4}$.

9. What part:

a. of $\frac{14}{163}$ is $\frac{13}{163}$? *b.* of $\frac{7}{13}$ is $\frac{4}{13}$? *c.* of $\frac{4}{177}$ is $\frac{3}{177}$?
d. of $\frac{5}{11}$ is $\frac{8}{11}$? *e.* of $\frac{14}{19}$ is $\frac{13}{19}$? *f.* of $\frac{23}{58}$ is $\frac{13}{58}$?

10. What part of 1 bushel is 3 pecks?

1 bu. = 4 pk. 3 pk. = $\frac{3}{4}$ of 4 pk. 3 pk. = $\frac{3}{4}$ of 1 bu.

11. What part:

a. of 1 yd. is 2 ft.? *b.* of 1 ft. is 1 in.? *c.* of 1 T. is 13 cwt.?

FRACTIONAL PARTS OF NUMBERS

1. What is $\frac{7}{8}$ of 5459?

First Method. $\frac{7}{8}$ of 5459 = 7 times $\frac{1}{8}$ of 5459.

$\frac{1}{8}$ of 5459 is $682\frac{3}{8}$, and 7 times $682\frac{3}{8} = 4776\frac{5}{8}$.

Second Method. $\frac{7}{8}$ of 5459 = $\frac{1}{8}$ of 7 times 5459.

7 times 5459 = 38,213, and $\frac{1}{8}$ of 38,213 = $4776\frac{5}{8}$.

First Method	$8\overline{)5459}$	*Second Method*	5459
	$682\frac{3}{8}$		7
	7		$8\overline{)38213}$
	$4776\frac{5}{8}$		$4776\frac{5}{8}$

2. What is the value of:

a. $\frac{6}{7}$ of 3843? *b.* $\frac{3}{5}$ of 7148? *c.* $\frac{67}{75}$ of 46,935?

d. $\frac{5}{8}$ of 7987? *e.* $\frac{11}{12}$ of 58,437? *f.* $\frac{137}{144}$ of 87,516?

FINDING NUMBERS FROM FRACTIONAL PARTS

3. $5861 = \frac{1}{7}$ of what number?

If $5861 = \frac{1}{7}$ of some number, then $\frac{7}{7}$ must equal 7 times 5861, which is 41,027.

Proof. $\frac{1}{7}$ of 41,027 = 5861.

4. Of what number:

a. does $3498 = \frac{1}{8}$? *b.* does $\frac{3}{4}\frac{4}{3}\frac{1}{8} = \frac{1}{9}$? *c.* does $59{,}387 = \frac{1}{9}$?

d. does $58.46 = .1$? *e.* does $\frac{1}{4}\frac{3}{9}\frac{7}{8} = \frac{1}{6}$? *f.* does $327.93 = .0001$?

THE EQUATION

An **equation** is an expression of equality between two quantities.

$7x = 14.$ $\quad 15\,(12 - 5 - 4) = x + 2\,(18 \div 9).$

5. If $8321 = \frac{1}{5}$ of a number, what is the number?

Let $x =$ the number $\quad \frac{x}{5} = 8321 \quad x = 8321 \times 5 = 41{,}605$

6. If $3497 = \frac{1}{9}$ of a number, what is the number?

FINDING NUMBERS FROM PARTS

1. $3476 = \frac{8}{9}$ of what number?

Let x = the number.

$$\frac{8}{9} \text{ of } x = 3476 \qquad \frac{8x}{9} = 3476 \qquad 8x = 9 \times 3476$$

$$x = \frac{9 \times 3476}{8} = 9 \times 434\tfrac{4}{8} = 9 \times 434\tfrac{1}{2} = 3910\tfrac{1}{2}$$

2. Of what number does:

a. $5496 = \frac{6}{7}$? *b.* $8372 = \frac{7}{10}$? *c.* $2475 = \frac{3}{10}$?

CHANGES IN THE TERMS OF A FRACTION

3. Reduce $\frac{3}{4}$ to an equivalent fraction, having 6 for a numerator.

The required numerator, 6, is 2 times the given numerator 3.

We multiply both terms by the same number.

$$\frac{2 \times 2}{4 \times 2} = \frac{6}{8}$$

4. Reduce $\frac{5}{7}$ to an equivalent fraction, having 10 for its numerator.

5. Reduce $\frac{6}{12}$ to an equivalent fraction, having 9 for its numerator.

6. Reduce $\frac{9}{10}$ to an equivalent fraction, having 6 for its numerator.

7. Reduce $\frac{2}{7}$ to twenty-firsts.

$$21 = 3 \times 7 \qquad \frac{2}{7} \times \frac{3}{3} = \frac{6}{21}$$

8. Reduce:

a. $\frac{3}{8}$ to twenty-fourths. *b.* $\frac{4}{5}$ to fiftieths. *c.* $\frac{2}{3}$ to ninths.
d. $\frac{4}{5}$ to thirtieths. *e.* $\frac{7}{9}$ to twenty-sevenths.
f. $\frac{8}{15}$ to forty-fifths. *g.* $\frac{4}{13}$ to sixty-fifths.
h. $\frac{6}{7}$ to forty-ninths. *i.* $\frac{7}{11}$ to twenty-seconds.

QUESTIONS

1. How many vest patterns, each containing $\frac{3}{4}$ yd., can be cut from a piece of vesting containing $25\frac{1}{4}$ yd.?

2. How many baskets, each holding $\frac{7}{8}$ of a peck, can be filled from $82\frac{3}{8}$ pecks of corn?

3. How many books, at $\$\frac{5}{6}$ each, can be bought for $\$232\frac{1}{6}$?

4. How many baskets, each holding $\frac{15}{32}$ of a bushel, can be filled from $453\frac{7}{32}$ bushels of apples?

5. How many barrels, holding $2\frac{1}{4}$ bushels each, can be filled from $59\frac{3}{4}$ bushels of apples?

6. What part of a ton of coal may be purchased for $\$5\frac{1}{16}$, at the rate of $\$6\frac{3}{4}$ a ton?

7. How long would it take a bicyclist to ride $125\frac{5}{7}$ miles, if he rode $11\frac{6}{14}$ miles an hour?

8. How many pieces of cloth, each piece $1\frac{2}{7}$ yards long, can be cut from a bolt of cloth containing $31\frac{1}{2}$ yards?

9. How long will it take 11 men to build a wall that 1 man can build in $15\frac{8}{9}$ days?

10. If $\frac{4}{5}$ of a barrel of flour cost $4\frac{7}{8}$, what is the price per barrel?

11. How many doz. eggs at $23\frac{1}{2}¢$ per doz. will pay for 10 lb. of butter at $21\frac{1}{4}¢$ per lb.?

12. If $\frac{3}{4}$ of $\frac{5}{9}$ of a farm is worth $2540, what is the value of the whole farm?

13. If $\frac{5}{6}$ of the value of a bicycle is $\$36\frac{1}{9}$, what is the bicycle worth?

14. If $\frac{5}{7}$ of $\frac{3}{4}$ of a pound of butter will last a family 1 day, how long will $9\frac{5}{14}$ pounds last them?

15. If 7 companies in a regiment muster 630 men, what will be the muster of 10 companies of similar size?

PROBLEMS

1. If $\frac{2}{3}$ of a vessel is worth \$880, what is the value of $\frac{3}{11}$ of the vessel?

2. A man has $\frac{3}{7}$ of a farm of 4000 acres left to him. He sells $\frac{2}{5}$ of his share. How many acres has he left?

3. If 16 horses eat 80 bushels of corn in 36 days, in how many days will 6 horses eat 45 bushels?

16 horses eat 80 bushels in 36 days.

Then 16 horses would eat 1 bushel in $(36 \div 80)$ days.

Also 1 horse would eat 1 bushel in $\frac{36 \times 16}{80}$ days.

And 6 horses would eat 45 bushels in $\frac{\overset{6}{\cancel{36}} \times \overset{1}{\cancel{16}} \times \overset{9}{\cancel{45}}}{\underset{1}{\cancel{6}} \times \underset{5}{\cancel{80}}}$ days,

$= 6 \times 1 \times 9 = 54$ days.

4. If 25 horses eat 100 bu. of corn in 30 days, in how many days will 500 horses eat 1000 bu.?

There are several different ways of finding the answers to **3** and **4**.

5. If 858 men in 6 months consume 234 barrels of flour, how many barrels will be consumed by 979 men in three months?

6. How many persons should share $1\frac{1}{2}$ lb. of candy so that each person has $\frac{1}{8}$ lb.? $\frac{1}{4}$ lb.? $\frac{1}{6}$ lb.?

7. What will $\frac{2}{5}$ T. of sugar cost at \$$4\frac{3}{8}$ per cwt.?

8. I bought sugar, and sold it at \$$\frac{1}{4}$ per bbl. advance, gaining \$180. How many barrels did I buy?

9. A has $88\frac{1}{2}$ ¢, B has $18\frac{3}{4}$ ¢. What part of B's money is A's money?

10. A corporation having its capital divided into 6000 shares, voted to represent its property by 20,000 shares. What part of an old share was a new one?

QUESTIONS

1. A train travels 25 mi. in $\frac{7}{10}$ hr. What is its rate per hour?

2. A bicycle rider went 45 mi. in 4 hr. What was his rate per hour?

3. If $\frac{2}{3}$ yd. of cloth cost \$3.74, what will 1 yd. cost?

4. If $\frac{5}{9}$ of a vessel cost \$17,645, what is its value?

5. If $\frac{5}{9}$ lb. of tea is worth \$$\frac{25}{27}$, what is a pound worth?

6. If $\frac{8}{9}$ yd. of silk cost \$1.572, what will $\frac{20}{27}$ yd. cost?

7. How much will $\frac{7}{12}$ cd. of wood cost, if $\frac{5}{9}$ cd. cost \$3.85?

8. If a ship sails 157 miles in $\frac{7}{12}$ of a day, how many miles will she sail in $8\frac{3}{4}$ days?

9. A man sold $8\frac{5}{12}$ tons of steel for \$384, and afterwards sold $9\frac{3}{8}$ tons more at the same rate per ton. How much did he receive for the second lot?

10. How much must be paid for $5\frac{5}{6}$ acres of land, when \$868 is paid for $2\frac{2}{3}$ acres?

11. If $6\frac{3}{8}$ bbl. flour cost \$38.25, how much will $14\frac{5}{9}$ bbl. cost?

12. If $\frac{7}{8}$ of a dollar will purchase $7\frac{7}{12}$ lb. of prunes, how many pounds can be purchased for \$$5\frac{1}{4}$?

13. What will $7\frac{3}{4}$ pounds of pepper cost, at \$$\frac{3}{5}$ a pound?

14. What is the value of $3\frac{4}{5}$ cd. of wood, at \$$12\frac{3}{5}$ per cd.?

15. How much must be paid for 18 handkerchiefs at \$$\frac{3}{4}$ each?

16. What cost $35\frac{4}{5}$ pounds of butter at $18\frac{3}{4}$¢ a pound?

17. Find the value of $212\frac{2}{3}$ lb. of veal, at $7\frac{1}{2}$¢ per lb.

QUESTIONS

1. If $1\frac{11}{20}$ T. of hay cost $\$22\frac{7}{8}$, how much will $9\frac{3}{5}$ T. cost?

2. I bought a bolt of silk for \$96, and sold it for $\frac{5}{8}$ of the cost, thereby losing \$2 per yard. How many yards were there in the bolt?

3. From a bolt of muslin containing $37\frac{3}{4}$ yd., there were cut at different times $9\frac{7}{8}$ yd., $6\frac{1}{2}$ yd., $3\frac{5}{8}$ yd., and $7\frac{2}{3}$ yd. How many yards of muslin were left?

4. What number subtracted from $14\frac{3}{8}$ will leave $1\frac{3}{4}$ for a remainder?

5. What will $7\frac{1}{2}$ pounds of fish cost at $5\frac{3}{4}¢$ a pound?

6. What will $6\frac{7}{8}$ yards of braid cost, at $12\frac{1}{2}¢$ a yard?

7. What will $7\frac{2}{3}$ yd. of flannelet cost, at $9\frac{3}{4}¢$ per yd.?

8. What will $12\frac{1}{2}$ pounds of tea cost, at $62\frac{1}{2}¢$ a pound?

9. What will 15 pounds of beef cost, at $9\frac{1}{2}¢$ a pound?

10. At $\$\frac{7}{8}$ a yard, what will $2\frac{3}{5}$ yards of cheviot cost?

11. At $3\frac{4}{5}¢$ a yard, how much will 15 rolls of braid cost, each roll containing 7 yards?

12. Find the value of $14\frac{4}{5}$ barrels of molasses, at $\$10\frac{2}{5}$ a barrel.

13. We may solve such questions as these in dollars and cents as well as in parts of dollars. In **13**, page 89, $\$\frac{3}{5} = 60¢$. $60¢ \times 7\frac{3}{4} = ?$ Or we may reduce the answer $\$4\frac{13}{20}$ to \$ and ¢. $\$4\frac{13}{20} = \$4.65$.

14. If the sum of two fractions is $\frac{5}{8}$, and one of the fractions is $\frac{9}{20}$, what is the other fraction?

15. A, B, and C buy a vessel for \$6300; A pays $\frac{3}{7}$ of the cost, B, $\frac{4}{9}$, and C the remainder. What did each one pay?

16. How many yards of carpet, $\frac{3}{4}$ of a yard wide, will cover a hall $20\frac{1}{2}$ yards long and $1\frac{1}{2}$ yards wide?

Draw to scale on paper or blackboard a picture to illustrate this.

17. What is the divisor if the dividend is $\frac{7}{9}$ and the quotient $\frac{4}{3\,9}$?

18. If a man earns $\$1\frac{4}{5}$ a day, how many days must he work to earn $\$36\frac{3}{5}$?

19. A person owning $\frac{3}{7}$ of property worth $2000, sold $\frac{2}{3}$ of his share. How many dollars' worth did he retain?

20. A man having $240, paid debts as follows: To A, $\frac{1}{3}$ of the money; B, $\frac{1}{10}$; C, $\frac{1}{8}$; and D, $\frac{1}{6}$. How many dollars did he pay A? B? C? D? How many dollars did he have left?

21. A speculator invested $3740 in copper-mining shares, and found at the end of 3 years that he had gained $\$156\frac{1}{3}$ more than $33\frac{1}{3}\%$ of his investment. How much did he gain yearly?

22. A and B engaged in a speculation. A furnished $\frac{7}{12}$ of the capital, and B the rest; if B had furnished $\$492\frac{2}{3}$ more, their shares would have been equal. What sum did each invest?

23. In one day a merchant made $63, which was $\frac{7}{8}$ of what he made the next day. $\frac{5}{3}$ of what he made in the second day was just 4 times what he made in the third day. How much did he make in the three days?

24. A cattle dealer had $\frac{1}{5}$ of his cattle in one pen, $\frac{1}{4}$ in another pen. The rest of the cattle, 77 in number, were in a third pen. How many cattle had he in the three pens?

25. What are $7\frac{3}{4}$ bu. of peaches worth at $\frac{1}{3}$ of $\$1\frac{4}{5}$ per bu.?

26. At $\$\frac{1}{4}$ a yard, how many yards of gingham can be bought for $\$5\frac{7}{8}$?

QUESTIONS

1. If a cord of wood cost $7.80, what will $\frac{3}{4}$ of a cord cost?

2. If a hogshead of molasses cost $17.84, what is the value of $\frac{7}{8}$ hhd.?

3. If $786.63 is paid for a kiln of brick, what should be paid for $\frac{11}{12}$ of a kiln?

4. If $3.52 is paid for $\frac{4}{5}$ of a ton of coal, what is the price per T.?

5. If $\frac{3}{10}$ cd. of wood cost $2.34, what will a cd. cost?

6. If $\frac{3}{4}$ yd. of silk cost $5.40, what is the value of $\frac{7}{9}$ yd.?

7. If $\frac{11}{15}$ of a house lot cost $665.50, what sum will pay for the lot?

8. When 15.57\frac{1}{2}$ is paid for doing $\frac{7}{8}$ of a piece of work, what will it cost to do all the work?

9. If 73.60\frac{5}{6}$ is paid for $\frac{11}{12}$ of a car of wheat, what sum must be paid for the whole car?

10. If $\frac{11}{12}$ of a pile of wood cost 80.20\frac{5}{6}$, what is the value of the whole pile?

11. I paid 1837\frac{5}{8}$ for 7350$\frac{1}{2}$ yards of batiste. What was its value a yard?

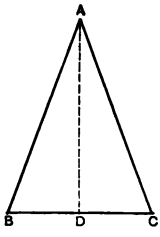
12. If 235$\frac{1}{2}$ acres of land cost 4725\frac{3}{8}$, how much will 628 acres cost at the same price an acre?

13. I owned $\frac{2}{5}$ of a mill and sold $\frac{1}{3}$ of my share for 5408\frac{3}{4}$. How much was the mill worth?

14. When $80 is paid for $\frac{3}{4}$ of a quantity of coffee beans, what will $\frac{7}{8}$ of the quantity cost?

15. If $\frac{4}{5}$ of a car of copper ore is worth $141.52, what is the value of $\frac{5}{2\,8}$ of a car?

TRIANGLES

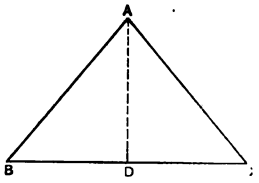


ABC is a triangle.

Its **altitude** is *AD*.

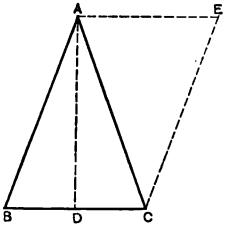
Its **base** is *BC*.

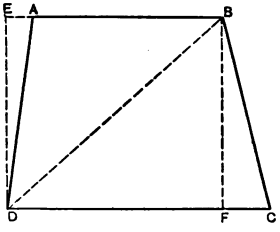
The area of a triangle $= \frac{\text{Base} \times \text{Altitude}}{2}$.



What is *AC* in relation to *ABCE*?

Cut out of paper the parallelogram *AECB*, and show that the triangles, *AEC* and *ABC*, are equal. What is the area of the parallelogram? What is the area of each triangle? What part of the parallelogram is either triangle?





ABCD is a **trapezoid**. Two of its sides are *parallel*.

ED or *BF* is its altitude.

AB is the base of the triangle *ABD*.

DC is the base of the triangle *DBC*.

The *area* of *ABCD* = *area* of *ABD* + area of *DBC*.

1. Draw triangles of various shapes with base 3 in. and altitude 4 in. Are all their areas equal?

2. Draw other triangles with base 6 in. and altitude 9 in. Find their areas.

3. Draw trapezoids and find their areas.

BILLS

A **bill of goods** is a written statement of articles sold, including the price of each article, the names of debtor and of creditor, and the amount of the indebtedness.

The **debtor** is the one who owes the debt, and the **creditor** is the one to whom the debt is owed.

A *bill* is receipted by writing the words "*Received payment*" or "*Paid*" at the bottom of the bill or across the face, with the signature of the *creditor* written beneath. If a clerk or other authorized agent writes the signature, he adds the word "Per" or "By," and his own initials.

1. Find the total amount of this bill:

ROCHESTER, N.Y., Aug. 1, 1900.

W. H. VICKS,

Bought of SAMUEL MARCY,

DEALER IN FINE GROCERIES.

2 lb.	Tea,	.80	
3 "	Coffee,	.30	
10 "	Sugar,	.06	
1 "	Pepper,	.40	
1 "	Chocolate,	.80	
10 bars	Soap,	.05	

Received Payment,

SAMUEL MARCY,

Per C. H. M.

2. Make out bills for groceries.
3. Make out bills for meats and provisions.
4. Make out bills for dry goods.

1. PHILADELPHIA, Oct. 2, 1900.

MRS. GEORGE CHASE,

Bought of JAMES MARSHALL, DEALER IN DRY GOODS.

8 yd. of Velvet,	$5.00	
10 " " Muslin,	.20	
12 " " Golf Plaid,	1.25	
3 " " Percale,	.40	
20 " " Dimity,	.50	

2. DETROIT, June 14, 1900.

THOMAS HALLECK,

Bought of WILLIAM HOLMES,

MEATS AND PROVISIONS.

6 lb. Sirloin,	.25	
10 " Roast Beef,	.15	
8½ " " Lamb,	.12	
15¼ " " Turkey,	.25	
4 " Suet,	.08	

Paid June 15, W. HOLMES, *By* R. H. S.

3. BANGOR, ME., June 29, 1900.

CHARLES FESSENDEN,

Bought of ISRAEL POTTER,

6500 ft. Hemlock Boards,	$6.00 per M.	
1050 " Sills,	5.00 " "	
8000 " Joists,	6.00 " "	
7200 " Pine Boards,	11.00 " "	

REVIEW

1. George Mitchell bought of Vaile & Tripp, New York, Apr. 14th, 1900, 3 lb. Bl. Tea, 65¢; 13 lb. Prunes, 11¢; Apr. 16th, 5 lb. Lard, 9¢; Apr. 18th, 3 bars Soap, 23¢; 3 boxes Starch, 15¢; Apr. 21, 1 Bath-brick, 8¢; 3 doz. Eggs, 17¢; 4 lb. Butter, 19¢; Apr. 23d, 12 lb. Flour, 3¢; 1 box Soda Biscuits, 30¢; Apr. 25th, 4 lb. Currants, 8¢, and 7 lb. Raisins, 9¢. On Apr. 21st the sum of $5 was paid on the above account, and the balance was paid on May 1st. Make out a receipted bill.

2. Received $2157, $2543, and $1626. Paid $312, $265, $763. What is the balance in hand?

	Dr.		*Cr.*	
	$2157		$312	Subtrahends
	2543		265	Subtrahends
	1626		763	Subtrahends
		Balance	4986	Remainder
	$6326		$6326	Minuend

3. Received $3162, $2154, $3172, and paid $2163, $84, $13; find the balance in hand.

4. Received $7000, and paid $213, $715, $610, $216; find the balance in hand.

If there are several amounts to be added, and several to be subtracted, the amounts may first be classified, and those of like kind added together. Then the difference of their sums may be found.

5. Received $25, paid $19, received $700, paid $16, $200, and $31, and received $15; find the balance.

6. At the beginning of a year a railroad had 19,615 cars. During the year 13 cars were smashed in accidents, 115 broken up, and 210 sold. 560 new cars were made. How many cars had the railroad at the end of the year?

7. A train going 45 miles an hour passed another going 36 miles an hour. How far apart would the trains be in two hours, if they were going in opposite directions?

REVIEW

1. $2\frac{5}{8}$ of $\frac{4}{5}$ of $5 = ?$

2. $\frac{1}{3}$ of $\frac{2}{5}$ of $2\frac{1}{2} = ?$

3. $\frac{5}{9} \times 3\frac{3}{4} \times 1\frac{4}{5} = ?$

4. $\frac{5}{9}$ of $\frac{8}{11}$ of $4 = ?$

5. A man gave away $10,561. He gave $\frac{2}{7}$ of it to a library, $\frac{1}{3}$ of the remainder to his brother, and the rest he divided equally among his 7 children. What sum did each of his children receive?

6. Find the value of:

a. $6 \div \frac{1}{2}$. *d.* $36 \div \frac{6}{11}$. *g.* $12 \div \frac{3}{5}$. *j.* $12 \div \frac{9}{5}$.

b. $12 \div \frac{4}{5}$. *e.* $6 \div \frac{2}{3}$. *h.* $42 \div \frac{28}{33}$. *k.* $8 \div \frac{2}{3}$.

c. $12 \div \frac{2}{5}$. *f.* $12 \div \frac{6}{5}$. *i.* $12 \div \frac{1}{5}$. *l.* $63 \div \frac{15}{22}$.

7. Divide:

a. .9375 by .075. *d.* 18 by .288. *g.* 15.875 by .0025.

b. 215.7 by 86.28. *e.* 25.2835 by 7.81. *h.* 7.3926 by 2.22.

c. 24.75 by .25. *f.* 11.83 by .845. *i.* 568.944 by 3.24.

8. Change: *a.* 5 to 8ths. *b.* 13 to 4ths. *c.* 8 to 32ds. *d.* 9 to 8ths. *e.* 5 to 5ths. *f.* 6 to 4ths.

9. Change: *a.* $\frac{5}{9}$ to 36ths. *b.* $\frac{3}{7}$ to 49ths. *c.* $\frac{6}{17}$ to 51sts.

10. There were 7178 A. of allspice or pimento berry plantations in Jamaica in 1898. The total yield of berries was 6,857,858 lb. What was the average yield per A.?

11. A Western farmer cut three crops of alfalfa from his meadows. The yield was 7 tons to the acre and was sold at $14.00 a ton. How much did he get for one season's crop on 12 acres?

12. Southern California, during the year 1899, exported to the Atlantic States 12,000 car loads of oranges. If each car contained 334 boxes, and each box held 75 pounds of fruit, how many pounds of oranges were exported?

ARITHMETIC

Arithmetic is essential to *business* and to *science*.

Arithmetic is the science and the art of measuring magnitudes and of counting numbers.

A **magnitude** is any thing.

A *line* has one dimension of magnitude, **length**.

A *surface* has two dimensions, **length** and **breadth**.

A *solid* has three dimensions, **length**, **breadth**, and **thickness**. *Thickness* may be height or depth.

A **number** counts things or units.

Magnitude tells, **How much?**

Number tells, **How many?**

Like numbers have units of the same magnitude or kind: three desks and five desks; seven ounces and two ounces; four boys and nine boys.

Unlike numbers have units of different magnitudes or kinds: ten feet and eight dollars; fifteen pounds and eleven hours.

A number is **concrete** when it is applied to one or more magnitudes or objects: forty dollars; twenty-three yards.

A number is called **abstract** when it is not applied to any magnitude or object: six, twelve, two hundred.

The relative value of the magnitude to the unit is its **ratio**. All ratios are abstract numbers.

The ratio of the magnitude ten acres to the unit one acre is the abstract number ten. The ratio of one dollar to the unit three dollars is one third. The ratio of seventeen dozen eggs to the unit one dozen is seventeen.

Notation is a method of writing numbers.

Numeration is a method of reading numbers.

We use commonly two kinds of notation, the Arabic and the Roman, but only one kind of numeration, the decimal, which is the basis of both kinds of notation.

TERMS AND PROCESSES

Addition finds the sum of two or more numbers.

The numbers to be added together are called **addends.**

The sign, +, *plus*, placed between two numbers means that the numbers are to be added together.

9 + 6, *nine plus six*, means, Add nine and six together.

1. Read: 7 + 5 + 10; 9 + 900 + 9,000,000; M + C + V.

The sign, =, read *equals* or *equal*, is called the sign of **equality.**

7 + 8 = 15, is read *seven plus eight equals fifteen.*

An expression of equality is termed an **equation.** The expression, 7 + 8 = 15, is an equation.

x may represent *anything*, or What? 10 + 12 = x.

5 + 6 + 8 = x.

Subtraction finds the difference between two numbers. Only **like numbers** can be added or subtracted.

The number from which another number is taken is the **minuend.**

The number taken from the *minuend* is the **subtrahend.**

The number resulting from *subtraction* is the **difference** or *remainder.*

The sign, −, *minus*, placed between two numbers means that the first number is the minuend and the second is the subtrahend.

9 − 5, *nine minus five*, means, From nine take five.

2. Read: 10 − 6; 18 − 13; 11,005 − 9985; D − CC.

3. Read and answer:

a. C + XL + IX − VII = ?

b. 250 + 150 + 75 + 125 + 400 − 1000 = ?

c. 500 + D − M + 70 + V = ?

d. 85 + 25 + 40 − 20 = 50 + x.

TERMS AND PROCESSES

Multiplication repeats one number as many times as there are units in another number.

The **multiplicand** is the number to be repeated.

The **multiplier** is the number which tells how many times the *multiplicand* is to be repeated.

The **product** is the number resulting from the *multiplication.*

The **factors** of a number are the numbers that, multiplied together, give the number.

The *multiplicand* and *multiplier* are *factors* of the *product.*

The sign of multiplication, $\times$, is read, *multiplied by.*

15×3 is read, *Fifteen multiplied by three.*

Division finds how many times one number is contained in another.

The number divided by another number is the **dividend.**

The number dividing another number is the **divisor.**

The number denoting *how many times* the dividend contains the divisor is the **quotient.**

The sign $\div$ is read, *divided by.*

$20 \div 5$ is read, *Twenty divided by five.*

The *divisor* and *quotient* correspond with the *factors* in *multiplication;* and the *dividend* corresponds with the *product.*

Division finds one of the two factors of a product when the other factor is known.

If the dividend does not contain the divisor exactly, we have either a *quotient* with a **remainder** or a *fractional quotient.*

To prevent confusion in processes it is better to call the result of subtraction the *difference* than to call it the *remainder.*

Multiplication is a short method of adding the same number to itself two or more times.

Division is a short method of subtracting the same number several or many times from another number.

Divisor × **quotient** = **dividend.**

Product ÷ **multiplier** = **multiplicand.**

The *multiplicand* and *product* are *like numbers.*

The *multiplicand* may be *abstract* or *concrete.*

The *multiplier* is always an *abstract* number.

The *product* is always of the same kind as the *multiplicand.*

The *dividend* and *quotient* are *like numbers.*

The *divisor*, *dividend*, and *quotient* may be *abstract* or *concrete.*

The multiplicand is always supposed to be in substance a number of things, or a concrete number. This is also true of the dividend.

In division by *partition* the divisor is abstract, or a ratio; but in division by *measurement* the divisor is concrete.

1. Why cannot we divide 9 qt. by 2 pt.?
2. Why cannot we multiply 15 yd. by 20 yd.?
3. Why cannot we add 10 cu. ft. to 3 cu. yd.?
4. What must we do to answer the problems above?
5. Divide 98 cu. ft. by 8.

A. 8)98 cu. ft.

12 cu. ft. + 2 cu. ft. *remainder.*

This is an inexact result.

B. 8)98 cu. ft.

$12\frac{2}{8}$ cu. ft. $= 12\frac{1}{4}$ cu. ft. This is an exact result with a fractional quotient.

In *B* the remainder, seen in *A*, is itself divided by 8, and appears in the fraction $\frac{2}{8}$.

PRECEDENCE OF SIGNS

Parentheses ( ), brackets [ ], braces { }, and the vinculum ——, when used to inclose numbers, are called **symbols of aggregation.** Two or more numbers inclosed by these symbols or covered by the vinculum are treated as single numbers. Operations indicated by signs between inclosed numbers must be performed before operations on numbers outside the symbols.

In any expression in which the signs $\times$, $\div$, $+$, $-$, occur, division and multiplication are performed before addition and subtraction, unless a symbol of aggregation indicates a different order of operations.

$$63 \div 7 + 2 = 9 + 2 = 11;$$

but $$63 \div (7 + 2) = 63 \div 9 = 7.$$

$$\frac{63}{7} + 2 = 9 + 2 = 11, \text{ and } \frac{63}{7 + 2} = \frac{63}{9} = 7.$$

When no sign occurs between the first parenthesis and the first number inclosed, the plus sign is regarded as standing before the number. In the equation $16 - (3 + 4) = 9$, the number 3 is considered as having the plus sign before it.

1. Compare $16 - (3 + 4) = 9$ with $16 - 3 + 4 = 17$.
2. Explain:

a. $5 \times 7 + 8 + 3 \times 2 = x$. *b.* $16 \div 4 + 8 \times 3 - 4 - 10 = x$.

3. Express by symbols and figures:

a. From 34 take the sum of 25 and 3.

b. To 30 add the difference between 25 and 8.

c. To the difference between 30 and 25 add 3.

4. To 10 add 6 and from the sum take 7 divided by 4.

ROMAN NOTATION

1. Read: I, V, X, L, C, D, M.

LXXIII	CXLIV	DXI
CLXIII	DCLXIV	CCCI
MDCCCLXXII	DCCCXLIII	XVIII
XLIX	MMDCCCLVIII	CCCXXX
CCXCVI	LX	XLVII
MCD	XXXIII	LXXIV
MMMXXX	XLIV	DCCLXXVII
XCII	XCIX	MI
CXCIX	CDLXXVIII	MDXXXIX
LXVIII	CMX	CCCXXXIII

2. A bar, called a *vinculum*, over a letter increases its value a thousand times. $\overline{V} = 5000$.

3. Read:

$\overline{X}$	$\overline{XC}$	$\overline{VM}$
$\overline{XX}$VMMXXVII	$\overline{XL}$CDXLIV	MDCCCXCIX
MDCCXCI	MCMXXXIII	$\overline{V}$CMLXXVII
MMCCXXII	$\overline{V}$MMMDCCCLXXXVIII	

4. Express in Roman notation:

202, 47, 91, 80, 20, 77, 101, 10, 111, 606, 437, 908, 899, 763, 497, 829, 827, 999, 888, 850, 2333, 3232, 3333, 4321, 1234, 5678, 8765, 9999, 25,671, 891,347, 912,342, 1,678,942, 16,713, 191,919, 29,134, 23,476, 912,345, 1,678,942, 3,456,713, 9,308,275.

5. The next to the last page in the preface of a book was numbered xxxiv. What should be the Arabic numerals upon the last page of the preface?

6. What Roman numerals are placed upon the corner-stones of buildings this year?

PERCENTAGE

per cent means hundredth or hundredths.

7 per cent = 7 hundredths.

$22\frac{1}{2}$ per cent = $22\frac{1}{2}$ hundredths.

The sign, %, is employed in commercial transactions to represent the words "per cent."

EQUIVALENTS

Words		Symbols		Decimals		Common Fractions		
1 per cent	=	1 %	=	.01	=	$\frac{1}{100}$	=	$\frac{1}{100}$
2 per cent	=	2 %	=	.02	=	$\frac{2}{100}$	=	$\frac{1}{50}$
3 per cent	=	3 %	=	.03	=	$\frac{3}{100}$	=	$\frac{3}{100}$
4 per cent	=	4 %	=	.04	=	$\frac{4}{100}$	=	$\frac{1}{25}$
5 per cent	=	5 %	=	.05	=	$\frac{5}{100}$	=	$\frac{1}{20}$
6 per cent	=	6 %	=	.06	=	$\frac{6}{100}$	=	$\frac{3}{50}$
7 per cent	=	7 %	=	.07	=	$\frac{7}{100}$	=	$\frac{7}{100}$
8 per cent	=	8 %	=	.08	=	$\frac{8}{100}$	=	$\frac{2}{25}$
9 per cent	=	9 %	=	.09	=	$\frac{9}{100}$	=	$\frac{9}{100}$
10 per cent	=	10 %	=	.10	=	$\frac{10}{100}$	=	$\frac{1}{10}$
20 per cent	=	20 %	=	.20	=	$\frac{20}{100}$	=	$\frac{1}{5}$
50 per cent	=	50 %	=	.50	=	$\frac{50}{100}$	=	$\frac{1}{2}$
100 per cent	=	100 %	=	1.00	=	$\frac{100}{100}$	=	1
125 per cent	=	125 %	=	1.25	=	$\frac{125}{100}$	=	$\frac{5}{4}$

$2\% = \frac{1}{50}$	$31\frac{1}{4}\% = \frac{5}{16}$	$62\frac{1}{2}\% = \frac{5}{8}$	$75\% = \frac{3}{4}$
$4\% = \frac{1}{25}$	$8\frac{1}{3}\% = \frac{1}{12}$	$87\frac{1}{2}\% = \frac{7}{8}$	$33\frac{1}{3}\% = \frac{1}{3}$
$5\% = \frac{1}{20}$	$41\frac{2}{3}\% = \frac{5}{12}$	$16\frac{2}{3}\% = \frac{1}{6}$	$66\frac{2}{3}\% = \frac{2}{3}$
$10\% = \frac{1}{10}$	$58\frac{1}{3}\% = \frac{7}{12}$	$83\frac{1}{3}\% = \frac{5}{6}$	$50\% = \frac{1}{2}$
$6\frac{1}{4}\% = \frac{1}{16}$	$12\frac{1}{2}\% = \frac{1}{8}$	$20\% = \frac{1}{5}$	$3\frac{1}{3}\% = \frac{1}{30}$
$18\frac{3}{4}\% = \frac{3}{16}$	$37\frac{1}{2}\% = \frac{3}{8}$	$25\% = \frac{1}{4}$	$2\frac{1}{2}\% = \frac{1}{40}$

1. What is 4 per cent of \$3? \$5? \$8? \$10? \$12?

2. What is 5 per cent of \$6? \$8? \$12? \$100?

3. What is 6 per cent of \$12? \$20? \$50? \$150?

4. What is 7 per cent of \$10? \$15? \$20? \$2500?

5. A farmer having a flock of 40 sheep lost 5 per cent of them. How many sheep had he left?

6. Express as decimal fractions: WRITE

a. 9 per cent.	*i.* $9\frac{3}{8}$ per cent.	*q.* $7\frac{1}{2}$ per cent.
b. $4\frac{1}{2}$ per cent.	*j.* 20 per cent.	*r.* $10\frac{2}{5}$ per cent.
c. $29\frac{1}{4}$ per cent.	*k.* 46 per cent.	*s.* $9\frac{5}{8}$ per cent.
d. $8\frac{1}{5}$ per cent.	*l.* 12 per cent.	*t.* $103\frac{1}{2}$ per cent.
e. $9\frac{3}{4}$ per cent.	*m.* 175 per cent.	*u.* 225 per cent.
f. $\frac{3}{8}$ per cent.	*n.* $6\frac{1}{4}$ per cent.	*v.* $\frac{3}{4}$ per cent.
g. $2\frac{5}{8}$ per cent.	*o.* $8\frac{3}{4}$ per cent.	*w.* $\frac{2}{3}$ per cent.
h. $\frac{1}{5}$ per cent.	*p.* $33\frac{1}{3}$ per cent.	*x.* $\frac{4}{5}$ per cent.

7. Express as common fractions in their lowest terms:

a. 22 per cent.	*c.* $31\frac{4}{5}$ per cent.	*e.* $27\frac{1}{3}$ per cent.
b. $28\frac{3}{8}$ per cent.	*d.* $51\frac{1}{4}$ per cent.	*f.* 325 per cent.

8. Write with the per cent sign, %:

a. .07. *b.* .61. *c.* 1.47. *d.* .06. *e.* 1.06. *f.* 1.11.

When the decimal fraction is smaller than .01, it may be represented by the sign % with the number before it expressed either decimally or by a common fraction.

$$.056 = 5\tfrac{6}{10}\% = 5.6\% = \text{also } 5\tfrac{3}{5}\%.$$

$$.0095 = \tfrac{95}{1000}\% = .95\% = \text{also } \tfrac{19}{20}\%.$$

9. Write with the sign, %:

a. .675.	*c.* 1.217.	*e.* .1355.	*g.* .2775.	*i.* .011.
b. .035.	*d.* .00125.	*f.* .0005.	*h.* 12.3.	*j.* .00575.

10. Admission for adults was 50¢, but for children it was 70% as much. What was it?

PERCENTAGE

1. A landlord sold a house for 20% more than its cost, $3000. What was the selling price?

First Method	$20\% = \frac{1}{5}$	*Second Method*	$3000
	$\frac{1}{5}$ of $3000 = $600		.20
	$3000 + $600 = $3600	*Gain*	$600.00

2. A dealer paid for goods $1825, and sold them so that he gained 12%. How much did he gain?

3. A lawyer collected $362.56, and charged 5% for his services. How much did he receive for his work?

4. A farmer having 1500 sheep, lost 25% of them. How many did he lose?

5. A man having $1960 in a bank, drew out 20% of it. How much had he left in the bank?

6. A dealer imported 1500 boxes of oranges, and $12\frac{1}{2}\%$ of them decayed. How many boxes did he lose, and how many had he left?

7. I bought a stock of goods for $4565, and paid $3\frac{1}{2}\%$ for freight. What was the whole cost of the goods?

8. A man's salary is $2000 a year, and he saves $37\frac{1}{2}\%$ of it. How much does he spend?

9. A man inherited $20,000 and spent 40% of it. How much had he left?

10. Two men engaged in business with $18,250 capital apiece; one gained 20% and the other lost 20% the first year. What was the capital of each man then?

11. What is the difference between 6% of $1040 and 7% of $905?

12. What per cent of $200 is $80?

13. What per cent of $1250 is $250?

USES OF PERCENTAGE

The principles of percentage are very helpful in **insurance, taxes, borrowing money, commissions, customs,** and **duties.** These are very important subjects in *business* and *government.*

1. If the tax rate in the city is $1\frac{7}{10}\%$, what is the amount of tax to be paid on a house assessed at $4000?

2. At the same rate what is the tax on a $9000 house?

3. The duty on silks was 40% of their value. What was the amount to be paid on $19,500 worth?

4. What is the annual premium on insurance for $39,000, at $2\frac{1}{5}\%$?

5. A farmer insures his barns and their contents to the amount of $17,800. What premium does he pay at $\frac{1}{2}\%$?

6. If the interest of $1 for 1 yr. is 6¢, what will be the interest of $5 for the same time? of $8? of $12? of $15? of $20? of $30? of $50? of $100? of $125?

7. If the interest of $1 for 1 yr. is 6¢, what will be the interest of $1 for 2 yr.? for 3 yr.? for 4 yr.? for 5 yr.?

8. A man took out a policy of life insurance for $5000 at $28 per $1000 premium. What was the amount of the premium? What per cent was this?

9. A farmer sent $1250 worth of produce to a merchant to sell at 5% commission. What was the amount of the commission, if all the produce was sold?

10. What is 12% of $1358?

11. What is 50% of $500,000?

12. What is .05% of $1,200,000?

Find out from your father and friends what you can about taxes, insurance, etc.

SYMBOLS OF DIVISION

Division may be expressed by placing the divisor *under* the dividend with a short line between them. The expression $\frac{72}{9}$ shows that 72 is to be divided by 9. It is read "72 divided by 9," or, seventy-two ninths.

1. Read the following expressions:

a. $54 \div 9 + 15 = 5 \times 3 + 6$.

b. $29 + \frac{15}{5} = 40 \div 5 \times 4$.

c. $48 \div 6 = 63 - 15 \div 12 \times 2$.

Division is also indicated by the sign); thus, 9)72 is equivalent to $72 \div 9$, or to $\frac{72}{9}$.

RATIO AND DIVISION

The quotient, which is the result of division, tells the **ratio** of the dividend to the divisor.

Division is also expressed by the ratio sign.

20 : 5 *twenty to five*, is equivalent to $20 \div 5$; $\frac{20}{5}$; or 5)20.

2. What is the ratio of 27 to 3? of 3 to 54?

3. What is the ratio of 7 to 11? of 11 to 14?

4. What is the ratio of 9 to 27? of 27 to 18?

5. What is the ratio of 42 to 5? of 5 to 84?

6. What is the ratio of 72 to 6? of 6 to 144?

7. Find the ratio of the following:

a. 5 to 25.	*b.* 19 to 152.	*c.* \$17 to \$8.50.
d. 47 to 7.	*e.* 23 to 299.	*f.* \$93 to \$31.
g. 83 to 7.	*h.* 147 to 21.	*i.* 14 bu. to 2 pk.
j. 187 to 11.	*k.* 28 to 35.	*l.* 24 lb. to 12 oz.
m. 244 to 2.	*n.* 1000 to 5.	*o.* 2 T. to 3 cwt.
p. 99 to 3.	*q.* 40 to 1000.	*r.* 2 cu. in. to 1 cu. ft.

RATIO

Two numbers having the same unit may be compared with each other in two ways:

By considering how much greater or less one is than the other; and

By considering how many times one contains the other.

Ratio is the relation which one number bears to another with respect to magnitude, when the numbers are compared by considering, not how much greater or less one is than the other, but how many times or parts of a time one contains the other.

The ratio of two numbers is the quotient arising from the division of one by the other.

The ratio of 18 to 6 is 3, since $18 \div 6 = 3$.

The ratio of 7 to 21 is $\frac{1}{3}$, since $7 \div 21 = \frac{7}{21} = \frac{1}{3}$.

Since ratio is a quotient, and since we have three ways of indicating division, it follows that we have three ways of expressing the ratio of one number to the other.

The ratio of 9 to 4 is expressed by $9 \div 4$, by $\frac{9}{4}$, and by $9 : 4$. The ratio of 7 to 13 is indicated by $7 \div 13$, by $\frac{7}{13}$, and by $7 : 13$.

Ratio can exist only between numbers of the same kind.

No comparison with respect to magnitude can be made between 6 hours and 11 pounds, or between \$19 and 16 miles.

The two given numbers which constitute the ratio are called the **terms** of the ratio; when spoken of together they are called a **couplet.**

The first term of a couplet is called the **antecedent**; the last term, the **consequent.**

Read: *a.* $5 : 14$. *b.* $21 : 6$. *c.* $100 : 1$.

Name the *antecedents* and the *consequents.*

PROPORTION

Proportion is an equality of **ratios.**

The ratios 15 : 3 and 25 : 5 constitute a proportion, since, 15 : 3 = 25 : 5.

The terms of the two couplets are called **proportionals.**

Proportion may be expressed in two ways:

By placing the sign of equality, =, between the ratios; and

By placing four dots, thus, : :, between the two ratios.

We may express the proportion existing between 15, 3, 25, and 5 by 15 : 3 = 25 : 5; or by 15 : 3 : : 25 : 5.

1. Read: *a.* 7 : 4 = 21 : 12. *b.* 9 : 15 : : 6 : 10.

a. Seven to four equals twenty-one to twelve.

b. Nine is to fifteen as six is to ten.

2. Read:

a. 12 qt. : 18 qt. = 18 qt. : 27 qt. *b.* 3 : 50 : : 12 : 200.

c. 100 mi. : $33\frac{1}{3}$ mi. : : 18 : 6. *d.* $x : 5 = 2 : 10$.

A ratio consists of two terms, an antecedent and a consequent. A proportion consists of two pairs of terms with the same ratio.

A proportion has two couplets, or four terms.

One ratio not in a proportion may be greater or less than another.

One proportion cannot contain one ratio greater or less than another, since equality does not admit of degrees.

The two outer terms of a proportion are called the **extremes,** and the two intermediate ones, the **means.**

In the proportion 3 : 17 = 21 : 119, 3 and 119 are the *extremes,* and 17 and 21 are the *means.*

In any *proportion*, the product of the *extremes* is equal to the product of the *means*.

$6 : 11 = 18 : 33$. Then $6 \times 33 = 11 \times 18$. $\frac{18}{33} = \frac{6}{11}$.

$6 : 11 = \frac{6}{11}$ and $18 : 33 = \frac{18}{33}$.

Reducing the ratios to a common denominator, we have,

$$\frac{6 \times 33}{11 \times 33} = \frac{18 \times 11}{33 \times 11}.$$

But since the fractions are equal and have the same denominators, their numerators must be equal:

$$6 \times 33 = 198 = 18 \times 11$$

But 6 and 33 are the extremes and 18 and 11 are the means; therefore, the product of the means equals the product of the extremes.

A proportion is an equation

$4 : 9 = 20 : 45$. $\frac{4}{9} = \frac{20}{45}$

If in a proportion any term is lacking, it may be found.

1. If the product of the means be divided by one extreme, the quotient will be the other extreme.

2. If the product of the extremes be divided by one mean, the quotient will be the other mean.

$$\text{1st term} = \frac{\text{2d term} \times \text{3d term}}{\text{4th term}}.$$

$$\text{2d term} = \frac{\text{1st term} \times \text{4th term}}{\text{3d term}}.$$

$$\text{3d term} = \frac{\text{1st term} \times \text{4th term}}{\text{2d term}}.$$

$$\text{4th term} = \frac{\text{2d term} \times \text{3d term}}{\text{1st term}}.$$

3. $x : 10 = 6 : 15$.

4. $x : 3 = 30 : 6$.

5. $2 : x = 22 : 99$.

6. $16 : x = 64 : 12$.

7. $2 : 15 = x : 90$.

8. $\frac{1}{2} : 6 = x : 3$.

9. $5 : 7 = 15 : x$.

10. $\frac{1}{5} : 3 = 7 : x$.

11. $x : \frac{2}{7} = \frac{7}{8} : \frac{3}{4}$.

12. $5\frac{1}{2} : x = 16 : 32$.

13. $\frac{5}{16} : \frac{1}{4} = 15 : x$.

14. $648 : 243 = 24 : x$.

PROPORTION

Proportion is sometimes called the **rule of three,** because when three terms are given, by means of them the fourth term may be found.

1. If 16 bbl. of flour cost $112, what will 129 bbl. cost?

$$\overset{\text{Barrels}}{16} : 129 = \overset{\text{Dollars}}{112} : x. \qquad x = \frac{129 \times 112}{16} = \$903.$$

Proof: Write 903 in the fourth place instead of x:

$$16 : 129 = 112 : 903.$$

Extremes: $16 \times 903 = 14{,}448.$

Means: $129 \times 112 = 14{,}448.$

2. If a man can walk 155 miles in 12 da., how many miles can he walk in 60 da.?

$$\overset{\text{Days}}{12} : 60 = \overset{\text{Miles}}{155} : x. \qquad x = \frac{\overset{5}{\cancel{60}} \times 155}{\underset{1}{\cancel{12}}} = 775 \text{ mi.}$$

Proof: $12 \times 775 = 9300 = 60 \times 155.$

3. If 11 baskets of peaches cost $13.42, what will 87 baskets cost?

4. If 28 cords of wood cost $266, what will 25 cords cost?

5. If a man receives $29.20 for 16 days' work, for how many days' work should he receive $83.60?

6. If 16 bu. of potatoes are sold for $12.80, what is the value of 156 bu.?

7. If a stick 7 ft. long casts a shadow of 5 ft., what is the height of a flagpole which casts a shadow of 112 ft. at the same time?

8. If a stack of hay will feed 27 cows for 99 da., how long will it feed 55 cows?

In solving problems by the method of proportion, think out very carefully how to state the proportion.

1. If a person can finish a journey in 100 da., traveling 10 hr. per day, how many days would he take if he traveled only 6 hr. per day?

2. What is the cost of 127 acres of land, if 871 acres cost $8671.40?

3. If a ship has water sufficient to last a crew of 35 men for 6 mo., how long will it last a crew of 20 men?

4. If the railway fare for 100 miles is $3.75, what should it be for 63 mi.?

5. If 16 bbl. of flour can be bought for $97.80, what should be paid for 27 bbl.?

6. What will 65 lb. of cheese cost if $1.30 pays for 13 lb.?

7. If 3 men build 73 rd. of fencing in 2 da., how long will they take to build 803 rd.?

8. If 176 pails of maple sap make 100 lb. of sugar, how much sugar will 1128 pails make?

9. If it costs $20.88 to buy 108 yd. of percale, how much must be paid for 465 yd.?

10. If $16 pay for the freightage of 720 barrels of flour, for how many barrels will $1278 pay the freightage?

11. If 11 men plow 165 acres in a week, how many acres will 3 men plow in the same time?

12. If 4 barrels of flour make 250 four-pound loaves of bread, how many such loaves will 67 bbl. make?

13. If 190 bu. of apples make 16 bbl. of vinegar, how many barrels of vinegar will 38 bu. of apples make?

14. If 90 men can build a wall in 12 da., how many men can build it in 15 da.?

FACTORING

An **exponent** is a figure written above and at the right of a number, and denotes how many times the latter is repeated as a factor; thus, $3^2 = 3 \times 3$; $7^3 = 7 \times 7 \times 7$; $2^5 = 2 \times 2 \times 2 \times 2 \times 2$.

1. Is 689 a prime number?

689 is not divisible by 2 because its last digit is an odd number, or by 3 or by 7 or by 11, but is divisible by 13; therefore, 689 is not a prime.

2. Is 947 a prime?

947 is not divisible by 2, 3, 7, 11, 13, 17, 19, 23, or 29; and when divided by the next prime, 31, the quotient is less than 31; therefore, 947 is a prime.

3. Find the prime factors of 44,856.

2	44856
2	22428
2	11214
3	5607
3	1869
7	623
	89

The number is divisible by 2, giving the quotient 22,428; 22,428 is divisible by 2, giving the quotient 11,214; 11,214 is divisible by 2, giving the quotient 5607; 5607 is divisible by 3, giving the quotient 1869; 1869 is divisible by 3, giving the quotient 623; but 623 is not divisible by 3.

Nor is 623 divisible by the next prime, 5; but it is divisible by 7, giving the quotient 89; and 89 is not divisible by 7. The square of 11, the next prime number, is 121, a number greater than 89; therefore, 89 must be prime. Hence,

$$44856 = 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 89 = 2^3 \cdot 3^2 \cdot 7 \cdot 89.$$

4. Find the prime factors of:

a. 12, 14, 15, 16, 18, 20, 22, 24, 46, 48, 49, 50, 51, 52.

b. 25, 26, 27, 28, 30, 32, 33, 34, 56, 57, 58, 60, 62, 63.

c. 35, 36, 38, 39, 40, 42, 44, 45, 54, 55, 64, 65, 66, 68.

5. Find the prime factors of:

a.	67,	69,	70,	72,	85,	87,	90,	95,	98.
b.	102,	111,	119,	125,	138,	146,	155.		
c.	154,	166,	178,	209,	234,	259,	267.		
d.	309,	366,	375,	404,	473,	524,	647.		
e.	6018,	76540,	63142,	78900,	6432,	97563,	89706.		
f.	7498,	56234,	49750,	3333,	99939,	48765,	92890.		

6. Resolve into their prime factors the numbers 29,656 and 15,725.

8	29656
11	3707
	337

29,656 is divisible by 8. The quotient is divisible by 11. The quotient is now 337, which is less than $400 = 20 \times 20$. Hence, unless 337 is prime, one of its factors is less than 20. No number under 12 divides it, and on trial with 13, 17, and 19 we find that 337 has no factor under 20.

Therefore $29,656 = 8 \times 11 \times 337$.

5	15725
5	3145
17	629
	37

15,725 can be divided by 5, and again by 5, giving the quotient 629. But 629 is less than 30×30, and its divisors are therefore less than 30. By trial with 13, 17, since it is divisible by no number under 12, we find 17 is contained 37 times in the number.

Therefore $15,725 = 5 \times 5 \times 17 \times 37 = 5^2 \times 37 \times 17$.

7. Resolve into their prime factors :

a.	1602.	*h.*	1827.	*o.*	555.	*v.*	2934.
b.	1632.	*i.*	36768.	*p.*	93555.	*w.*	32320.
c.	1365.	*j.*	851.	*q.*	30012.	*x.*	1011.
d.	15296.	*k.*	528.	*r.*	333333.	*y.*	707.
e.	378.	*l.*	1485.	*s.*	1118.	*z.*	19061.
f.	9625.	*m.*	13237.	*t.*	11368.	*a'.*	20625.
g.	616.	*n.*	15296.	*u.*	1024.	*b'.*	49560.

CANCELLATION

Cancellation shortens division by rejecting equal factors from both divisor and dividend.

Dividing divisor and dividend by the same number does not alter the quotient.

To reject a factor of any number is to divide the number by that factor.

When a factor is canceled, the unit 1 is considered to take its place.

One factor in the dividend will cancel only one equal factor in the divisor.

When all the factors or numbers in the dividend are canceled, the factor 1 should be retained.

1. Multiply 210 by 36 and divide the product by 60.

$$\frac{\overset{42}{\cancel{210}} \times \overset{3}{\cancel{36}}}{\underset{5}{\cancel{60}}} = \frac{42 \times 3}{1} = \frac{126}{1} = 126$$

We remove the factor 12 from the 36 in the dividend, and from the 60 in the divisor.

Dividing by 5 the quotient, 5, in the divisor and the factor 210 in the dividend, we have left in the dividend the factors 42 and 3. Their product is 126. 1 is the only factor left in the divisor. The quotient is $\frac{126}{1} = 126$.

2. $\dfrac{10 \times 15 \times 16}{25 \times 24} = ?$

3. $\dfrac{27 \times 28 \times 30}{35 \times 36 \times 18} = ?$

4. $\dfrac{54 \times 55 \times 56}{35 \times 66 \times 72} = ?$

5. $\dfrac{18 \times 20 \times 21}{28 \times 30} = ?$

6. $\dfrac{27 \times 32 \times 36}{21 \times 36 \times 40} = ?$

7. $\dfrac{55 \times 56 \times 60}{25 \times 32 \times 77} = ?$

8. What is the value of $36 \times 42 \times 52$ divided by $2 \times 3 \times 4$? divided by 9×21? divided by 21×13?

9. Which of the following numbers are prime? Find the prime factors of those which are composite:

a.	37,	4393.	93,	101,	7771,	111,	227.
b.	107,	9959,	113,	110,	1793,	207,	469.
c.	239,	7111,	349,	299,	713,	169,	3029.
d.	507,	6539,	2197,	2327,	581,	4573,	1363.

10. Divide the product of 45 times 84 by 9.

11. $15 \times 7 \times 12$ by $5 \times 3 \times 7 \times 2$.

12. $75 \times 15 \times 24$ by $25 \times 3 \times 6 \times 4 \times 5$.

13. What is the value of $99 \times 360 \times 365$ divided by 11×73? divided by 33×18? divided by 44×5?

14. What is the value of $9 \times 40 \times 100$ divided by $6 \times 8 \times 10$? divided by $3 \times 6 \times 25$?

15. $\dfrac{22 \times 24 \times 25}{33 \times 40} = ?$

16. $\dfrac{33 \times 35 \times 36}{27 \times 28 \times 55} = ?$

17. $\dfrac{56 \times 64 \times 65}{40 \times 32 \times 26} = ?$

18. $\dfrac{27 \times 35 \times 40}{45 \times 56} = ?$

19. $\dfrac{35 \times 40 \times 42}{25 \times 48 \times 49} = ?$

20. $\dfrac{64 \times 65 \times 70}{50 \times 52 \times 56} = ?$

21. $\dfrac{24 \times 28 \times 30}{36 \times 42} = ?$

22. $\dfrac{44 \times 45 \times 49}{28 \times 33 \times 35} = ?$

23. $\dfrac{70 \times 72 \times 80}{56 \times 60 \times 60} = ?$

24. $\dfrac{30 \times 32 \times 33}{44 \times 48} = ?$

25. $\dfrac{45 \times 48 \times 50}{25 \times 36 \times 60} = ?$

26. $\dfrac{75 \times 80 \times 81}{45 \times 45 \times 60} = ?$

27. $\dfrac{28 \times 30 \times 36}{42 \times 45} = ?$

28. $\dfrac{54 \times 56 \times 60}{42 \times 45 \times 48} = ?$

29. $\dfrac{80 \times 91 \times 90}{45 \times 56 \times 130} = ?$

30. $\dfrac{1000000}{8 \times 40 \div 2 + 5} = ?$

31. 11 doz. suits were sold at \$9.75 a suit in equal lots to 22 different men. How much in all did each pay?

QUESTIONS

1. A besieged town containing 15,000 inhabitants has provisions sufficient to last 5 weeks. How long will the provisions last if 7000 of the inhabitants be sent away?

2. How far can a man travel in 27 da. at the rate of 149 mi. in 4 da.?

3. If a person steps 4 yd. in 5 paces, to how many yards will 729 of his paces be equal?

4. @ 29¢ per lb., what is the cost of 174 lb. of coffee?

5. The numbers 24, 28, 32, 49, 81, are to be multiplied together and the product is to be divided by $8 \times 4 \times 7 \times 9 \times 6$. What will be the quotient?

6. Divide $16 \times 20 \times 8 \times 3$ by $30 \times 8 \times 6$.

7. How many pounds of raisins at 15¢ a pound are equal in value to 25 pounds of pepper at 48¢ a pound?

8. How much muslin at 25¢ a yard is equal in value to 100 yd. of silk at 87¢ a yard?

9. How many yards of serge at 46¢ a yard should be given for 23 bu. of wheat at 92¢ a bushel?

10. How long will it take a train to go 100 mi. at the rate of 42 mi. per hr.?

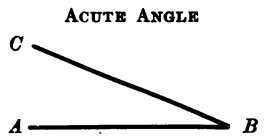
11. Add:

a.	*b.*	*c.*	*d.*	*e.*	*f.*	*g.*	*h.*
29	88	42	93	75	69	184	103
16	37	15	66	28	37	216	405
27	25	56	81	79	19	135	764
61	34	32	13	11	61	320	573
19	36	91	39	99	36	413	127
76	44	72	87	13	14	101	205
23	26	13	46	28	42	204	495
17	19	21	65	43	75	302	207
38	60	43	53	24	32	401	185

SURFACE MEASURE

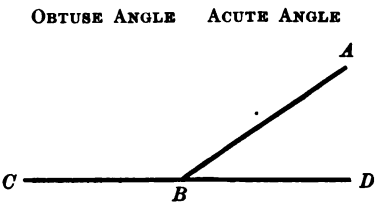
When two lines meet at a point, their difference in direction is called the **angle** of the two lines. The point where they meet is called the **vertex** of the angle.

The lines BA and BC form an angle whose vertex is at B.



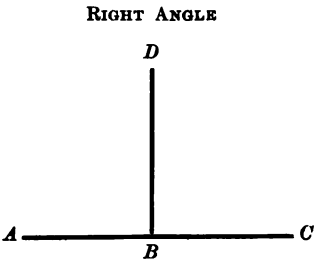
In reading an angle, the letter at the vertex is always made the middle one. The angle may be read either as the angle ABC or as the angle CBA.

In this figure there are two angles, DBA and ABC. The first is the difference in direction of the two lines AB and BD, and the second is the difference of the two lines AB and BC. Which angle is the larger?



When the two angles formed by one straight line meeting another straight line are equal to each other, they are called **right** angles, and the two lines are said to be **perpendicular** to each other.

When a straight line BD meets a straight line AC so as to make the **adjacent** angles ABD and BDC equal to each other, then ABD and DBC are each right angles, and DB and AC perpendicular to each other.



An angle greater than a right angle is called an **obtuse** angle, and one less than a right angle is called an **acute** angle.

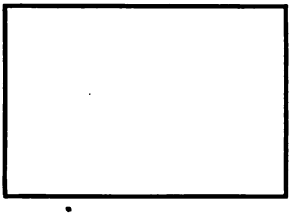
SURFACES

A **plane** has two dimensions, **length** and **breadth.**

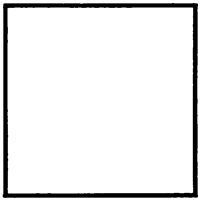
A four-sided plane figure, with all its angles right angles, is called a **rectangle.**

A rectangle, with all its sides equal, is called a **square.** A square has four equal sides and four equal angles.

RECTANGLE



SQUARE



If a square measures a foot on each side, it is called a square foot; if it measures a yard on each side, it is called a square yard. A square unit is any surface equivalent to a square one unit long and one unit wide.

All surfaces are measured by **square measure,** that is, by the number of squares of a given size to which the surfaces are equivalent.

The sum of the lengths of all the sides of any figure is its **perimeter.**

1. Measure the lengths of the perimeters of the two figures on this page.

2. Measure the length of the sides of the cover of this book.

3. How many sq. in. are in a square whose perimeter is 16 in. long? Draw the square and prove the answer.

4. How many sq. in. are in a rectangle whose perimeter is 24 in. long? Draw the figure on the blackboard.

REVIEW

1. In how many days can 54 men do the work that 1 man can do in 1198 days?

2. If a man's income is $5110 a year, how much is it a day? a week? a month?

3. If a steamer goes in 6 days from an Atlantic port to a port in Western Europe, 3070 miles, how many miles does the steamer go daily?

4. A man bought 156 pounds of prunes for $10.92, and sold them for 9¢ a pound. How much did he gain?

5. The product of two numbers is 51,679,780, and one of the factors is 615. What is the other factor?

6. A dealer has 5100 pounds of butter, and wishes to pack it in 60 tubs. How much must he put in each tub?

7. X bought a number of horses at $74 each, paying $17,484 in all for them. How many horses did he buy?

8. Find the value of:

a. $1\frac{3}{4}-1\frac{1}{9}$. *b.* $2\frac{1}{4}-\frac{7}{15}$. *c.* $7\frac{1}{3}-6\frac{7}{8}$. *d.* $84\frac{7}{15}-16\frac{1}{3}$.
e. $4\frac{1}{7}-\frac{3}{9}$. *f.* $9\frac{1}{2}-3\frac{7}{8}$. *g.* $246\frac{3}{8}-164\frac{1}{5}$. *h.* $16\frac{5}{8}-5\frac{8}{9}$.

9. Since 63 gallons make a hogshead, how many hogshead do 2898 gallons make?

10. A California orchard of 700 A. yielded in one year the following quantities of fruit: apricots, 339,411 lb.; peaches, 2,115,314 lb.; nectarines, 210,518 lb.; pears, 280,124 lb.; plums, 4705 lb.; prunes, 22,283 lb. What was the average number of pounds of fruit per A.?

11. A sold 4 T. of hay @ $12 per T., 80 bu. of wheat @ $1 per bu., and took in payment a cow worth $65, a wagon worth $40, and cash. How much did he receive?

12. Draw a triangle with base 7 in. and altitude $4\frac{1}{2}$ in. Find its area.

CIRCULAR MEASURE

Circular or **angular measure** is used to measure angles and the circumferences of circles.

A **circle** is a surface bounded by a curved line which is everywhere equally distant from a point within called the **center.** The perimeter or boundary line is called the **circumference** of the circle.

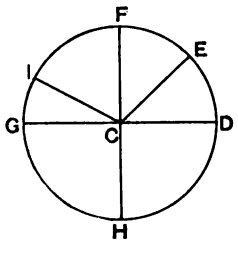
The distance from the center of a circle to the circumference is called the **radius** of the circle.

If from the vertex of an angle, as a center, we should draw a circumference, some portion of that circumference would be included between the sides of the angle. The larger the angle is, the larger will be the arc included between its sides, and the smaller the angle is the smaller will be the included arc.

Any portion of the circumference is called an **arc.**

Since the angle and the arc thus vary with each other, the arc is taken as the measure of the angle.

Draw angles and arcs and cut out paper to show this.



In this figure let C be the center of the circle and the vertex of the several angles. Let FH and CD be lines perpendicular to each other, and CE and CI be any other radii. Then the arc ED is the measure of the angle ECD; the arc DF, of the angle DCF; the arc DI, of the angle DCI; the arc EF, of the angle ECF, etc.

The line from a point on one side of a circle through the center to a point on the opposite side is called the **diameter** of the circle.

The circumference of any circle, may be divided into 360 equal parts, called **degrees.**

A degree is regarded as the 360th part of the circumference of a circle. Its actual length and that of its subdivisions vary with the size of the circle.

Since *FH* and *GD* are perpendicular to each other, the angles *FCD*, *DCH*, *HCG*, and *GCF* are all right angles, and each of them must include $\frac{1}{4}$ of the angular space about the point *C*. The arc included between the sides of a right angle equals $\frac{1}{4}$ of the circumference, and the measure of a right angle is $\frac{1}{4}$ of 360 degrees, or 90°.

The arc of an angle of 90° is called a **quadrant.**

Four quadrants make a circle. $90° \times 4 = 360°$.

Quadrant is the name both of the arc of 90° and of the surface included between the *arc* and the *angle* of 90°.

An *acute* angle is always less than 90°.

An *obtuse* angle is always more than 90°.

1. What part of a circumference is an angle of 45°?

2. What part of a quadrant is an angle of 60°?

3. Add together these angles:

a. 15°, 25°, 50°. What part does this equal?

b. 60°, 50°, 30°, 10°. What part of a circle is each?

c. 20°, 60°, 40°. What part of a circle is each?

4. Divide 90° into 3 equal angles.

5. Is an obtuse angle more or less than a quadrant?

6. Draw a circle with a diameter of 2 in.; another with a diameter of 4 in.; and another with a diameter of 6 in.

7. In each of the circles in 6 draw an angle of 30° and an angle of 150°.

8. How many degrees are there in $\frac{1}{2}$ a circumference?

Use blackboard drawings and paper-cuttings to illustrate these facts.

POWERS OF NUMBERS

When a number is multiplied by itself once, twice, three, four times, the resulting products are called the second, third, fourth **powers** of the number.

The term "square" is usually employed instead of second power. The term "cube" is usually employed instead of third power.

144 is the square of 12, because $12 \times 12 = 144$.

64 is the cube of 4, because $4 \times 4 \times 4 = 64$.

81 is the fourth power of 3, because $3 \times 3 \times 3 \times 3 = 81$.

1. Find the squares of:

15	24	40	57	69	72	87	100
114	237	625	897	371	225	634	1000
32	65	56	90	125	200	500	666

2. Find the cubes of:

9	11	25	47	68	93	257	48
10	50	75	100	200	300	400	1000

3. Find the fourth power of:

2	3	4	5	6	7	8	9
10	12	15	18	20	25	30	50

4. Find the tenth power of:

2	3	4	5	6	7	9	10

5. What do you notice about the higher powers of any number? Make a drawing to show the powers of 3.

6. Read: 6^5, 8^4, 25^3, 100^9, 10^{25}.

7. Find: 4^5, 10^5, 16^3, 2^8, 3^6, 5^9.

8. $4^4 \div 5^2 = ?$ Answer in decimals.

9. $2^3 \div 3^2 = ?$ Answer in common fractions.

10. $100^2 \div 5^3 = ?$ Answer in decimals.

THE EQUATION

The **first member** of an **equation** is the part on the left of the sign =. The **second member** of an equation is the part on the right of the sign =.

Since an equation is an expression of equality between two quantities, the following changes can be made in the members of an equation without affecting their equality:

The same or equal quantities may be added to both members.

$$15 + 3 = 3 \times 6. \quad 15 + 3 + 5 = 3 \times 6 + 5.$$

The same or equal quantities may be subtracted from both members.

$$8 \times 2 = 4 \times 4. \quad 8 \times 2 - 6 = 4 \times 4 - 6.$$

Both members may be multiplied by the same or equal quantities.

$$5 = 3 + 2. \quad 5 \times 2 = (3 + 2) \times 2.$$

Both members may be divided by the same or equal quantities.

$$10 = 8 + 2. \quad 10 \div 2 = (8 + 2) \div 2.$$

Transposition changes terms from one member of an equation to the other without changing the equality.

Every + or − term transposed must have its sign changed to the opposite. $8 + 3 + 1 = x + 5$. Take 5 from each member. $8 + 3 + 1 - 5 = x$.

1. Find the value of x in the equation $3x - 4 = 14$.

Adding 4 to both members, $3x - 4 + 4 = 14 + 4$.

$$3x = 18.$$

Dividing by 3, $x = 6$.

2. $x + 5 = 2(8 \div 4) + 7$. What is x?

3. $x - 5 - 3 = (2x + 4) \div 12$. What is x?

AN UNKNOWN QUANTITY

1. If 6 be taken from twice a certain number, the remainder will be 4. Find the number.

ARITHMETICAL EQUATION

2 times a certain number $- 6 = 4$.

2 times a certain number $= 4 + 6 = 10$.

The number $= 10 \div 2 = 5$.

ALGEBRAIC EQUATION

Let $x =$ the number.

$$2x - 6 = 4.$$

$$2x = 4 + 6 = 10.$$

$$x = 10 \div 2 = 5.$$

Discuss each step in the process.

2. A purse and the money it contains are worth \$24, and the money is worth 11 times the value of the purse

Let $x =$ value of purse.

$$x + 11x = 24.$$

$$12x = 24.$$

$$x = 24 \div 12 = 2.$$

Solve the following equations:

3. $2x+3x+4x=18$.

4. $6x+3x+9x=108$.

5. $12x-4x-3x=20$.

6. $9x-3x=18-6$.

7. $17x-114=198-7x$.

8. $17-x=2x-1$.

9. $12x-8=3x-98$.

10. $7x+9=4x+12$.

11. $6+4x=3x+9$.

12. $x+5=3x-9$.

13. $24x-49=19x-14$.

14. $4x-22=34-3x$.

15. $9x-3(5x-6)=-30$.

16. $26-8x=80-14x$.

17. $4x-(6x-35)=5x-(3x-7)$.

18. If 1 apple costs x¢, what will represent the cost of 2 apples? of 3? of 5? of 7?

19. If 1 lemon costs $2x$¢, what will represent the cost of 2 lemons? of 3 lemons? of 5? of 6?

20. I bought a number of apples at 2¢ apiece, and as many more at 3¢ apiece. If x represents the number of apples at 2¢, what will represent their cost? What will represent the cost of the apples at 3¢ apiece?

21. I bought lemons and oranges, of each an equal number; the lemons cost 2¢, and the oranges 3¢ apiece; the cost of the whole was 30¢. How many oranges were there? How many lemons?

22. I bought 2 lemons and 3 oranges for 16¢; the price of an orange was twice that of a lemon. If x represents the cost of a lemon, what will represent the cost of an orange? What will represent the cost of 2 lemons? of 3 oranges? of 2 lemons and 3 oranges? What was the cost of each fruit?

23. Fred has 4¢, and Tom has 1¢ less than Fred. How many cents has Tom?

24. If $3x - 2$ is equal to 10, to what is $3x$ equal? If $3x$ is equal to 12, to what is x equal?

25. If $5x - 3$ is equal to 17, to what is $5x$ equal? If $5x$ is equal to 20, to what is x equal?

26. If $x \div 2$ is equal to 4, to what is x equal?

27. If $x \div 3$ is equal to 5, to what is x equal?

28. If $2x \div 3$ is equal to 8, to what is x equal.

29. A dealer sold a horse for $100, a gain of 25%. What did it cost him?

SUGGESTION. — Let x = the cost to the dealer.

30. A boy bought a baseball for $1.25, which was 140% of its wholesale value. Find that value.

INTEREST

Interest is money paid by a borrower to a lender for the use of money. It is estimated as a certain per cent of the sum lent.

1. What is the interest of $100 for 1 yr. at 6%?

The interest of $100 for 1 yr. at 6% is $\frac{6}{100}$ of $100 = $6.

$100 = principal.
.06 = rate.
$6.00 = interest.

2. What is the interest of $611.35 for 3 yr. at 7%?

$611.35 = principal.
.07 = rate.
$42.7945 = interest for 1 yr.
3
$128.3835 = interest for 3 yr.

SIX PER CENT INTEREST

In a majority of the States the legal rate of interest is 6%. Hence it is very convenient to have a short method of computing interest at that rate.

At 6% a year the interest on $1

for 12 months	. . . . .	is	$.06.
" 2 mo. = $\frac{2}{12}$ of 12 mo. = $\frac{1}{6}$ of 12 mo.		"	.01.
" 1 mo., or 30 da. = $\frac{1}{12}$ of 12 mo.	.	"	.00½ = $.005.
" 6 da. = $\frac{1}{5}$ of 1 mo. or 30 da.	.	"	.001.
" 1 da. = $\frac{1}{6}$ of 6 da. = $\frac{1}{30}$ of 30 da.	.	"	.000$\frac{1}{6}$

We find the interest on $1 for any number of months by dividing the number of months by 2. The quotient is the interest in cents.

1. What is the interest of $1 for 8 months at 6 per cent?

8 ÷ 2 = 4. $.04.

2. What is the interest of \$1 for 7 years 3 months at 6 per cent?

7 years 3 months = 87 months, and $87 \div 2 = 43\frac{1}{2}$, \$.435.

3. What is the interest of \$1 for 11 years 7 months at 6 per cent?

4. Find the interest at 6% on \$1 for:

a. 6 mo.	*g.* 7 mo.	*m.* 3 yr.	*s.* 3 yr. 8 mo.
b. 3 mo.	*h.* 5 mo.	*n.* 6 mo.	*t.* 3 yr. 11 mo.
c. 9 mo.	*i.* 2 mo.	*o.* 4 yr.	*u.* 2 yr. 2 mo.
d. 1 yr.	*j.* 1 mo.	*p.* 6 mo.	*v.* 3 yr. 10 mo.
e. 2 yr.	*k.* 4 mo.	*q.* 1 yr. 6 mo.	*w.* 4 yr. 6 mo.
f. 8 mo.	*l.* 3 yr.	*r.* 2 yr. 3 mo.	*x.* 4 yr. 10 mo.

5. What is the interest of \$366.20 for 3 yr. 8 mo. at 6%?

3 yr. 8 mo. = 44 mo. $44 \div 2 =$ \$.22, int. on \$1.

\$366.20
.22
732 40
7324 0
\$80.56 40, int. on \$366.20 for 3 yr. 8 mo.

6. Find the simple interest on:

a. \$197 for 1 yr.	*e.* \$1728 for 1 yr. 6 mo.
b. \$1728 for 3 yr.	*f.* \$16.87 for 1 yr. 8 mo.
c. \$69 for 2 yr.	*g.* \$118.15 for 2 yr. 6 mo.
d. \$97.16 for 1 yr. 5 mo.	*h.* \$789.87 for 1 yr. 11 mo.

7. A drew his note payable to B for \$1575 at 6% a year, and paid it in 2 yr. 5 mo. How much interest was due then?

In *business* interest is usually paid on long term notes, those for one year or more, annually or semi-annually. If the interest due is not paid, interest on interest, compound interest, is estimated on debts.

INTEREST

We find the interest at 6 % for any number of days by dividing the number of days by 6. The quotient will be the interest in mills.

1. What is the interest of $1 for 18 days at 6 per cent?

$$18 \div 6 = 3\,(\text{mills}) = \$.003.$$

2. What is the interest of $1 for 26 days at 6 per cent?

3. Find the interest at 6% on $1 for:

a. 30 da	*e.* 24 da.	*i.* 3 da.	*m.* 28 da.
b. 15 da.	*f.* 23 da.	*j.* 9 da.	*n.* 27 da.
c. 6 da.	*g.* 18 da.	*k.* 16 da.	*o.* 21 da.
d. 20 da.	*h.* 12 da.	*l.* 14 da.	*p.* 10 da.

4. What is the interest of $1 for 7 yr. 4 mo. 27 da. at 6 per cent?

7 yr. 4 mo. = 88 mo., and $88 \div 2 = 44$ (cents) = $.44

= interest for 7 yr. 4 mo.

$27 \div 6 = 4\frac{1}{2}$ (mills) = $.0045 = interest for 27 da.

$.4445 = interest for 7 yr. 4 mo. 27 da.

5. Find the interest on $1 at 6 per cent a year for:

a. 16 years 4 months.
b. 3 years 3 months 3 days.
c. 5 years 11 months.
d. 4 years 7 months 10 days.
e. 11 years 1 month.
f. 1 year 9 months 25 days.

We can find the interest at 6% on any sum of money by finding the interest on $1 for the period and multiplying this by the number of dollars in the given principal.

6. Find the interest at 6% on:

a. $250 for 3 mo. 10 da.
b. $175 for 4 mo. 20 da.
c. $560 for 8 mo.
d. $1000 for 1 yr. 2 mo. 3 da.
e. $5,000,000 for 1 yr. 3 mo. 18 da.

QUESTIONS

1. Give the multiplication tables, 2 to 12.
2. What are the three dimensions of a *solid?*
3. What is a *cube?* a *circle?* a *triangle?*
4. Discuss *area.*
5. How thick is a pane of glass?
6. How high is an ordinary flour barrel?
7. What is the height of a horse?
8. What is meant by *perpendicular?*
9. What is an ordinary height for a ceiling in a house? for a chair?
10. What is an ordinary height for men?
11. How far can a boy jump who is a good jumper?
12. In what time can a good runner run a mile?
13. Of what use to boys and girls are the facts of weights and measures?
14. What amounts of money can boys and girls earn at different kinds of work?
15. What amounts of money do men earn?
16. What do we mean when we say that a man is a *millionaire?* Explain.
17. What is *property?* Why do we measure property by *money?* What is money?
18. How high is an oak? a flag pole? a telegraph pole?
19. What do you know about the heights of *mountains?* depths of *mines?* areas of *farms?* widths of *rivers?*
20. How long is a railroad car? a locomotive engine?
21. Are houses worth more than factories? Discuss.
22. Of what use to a boy or a girl is Arithmetic?
23. What are some of the uses of Arithmetic to men?

REVIEW

1. A prize captured by two United States cruisers in the Spanish-American war was valued at one hundred eighty-four thousand seven hundred seventy-five dollars. If this sum had been divided equally among the four hundred seventy-five men, composing the crews of both vessels, what would have been the share of each man?

2. What is the ninth part of $972?

3. A canal 126 miles in length cost $3,860,000. What was the cost per mile?

4. Find $\frac{3}{4}$ of $\frac{8}{45}$ of $\frac{24}{25}$.

5. A cubic foot of distilled water weighs 1000 ounces. What is the weight of one cubic inch?

6. If one man can do a piece of work in 504 days, how many days will it take 24 men to do the same work?

7. If an iceberg moves 6 miles a day, how many days will it take to move 2556 miles?

8. If in 178 books there are 56,070 pages, how many pages are there on an average in a book?

9. If 37 barrels of apples are sold for $55.50, what is the cost of a barrel?

10. Three partners began business with a capital of $1793; the first had $714, the second had $417. What did the third have?

11. After selling 1,016,085 oranges a merchant had two lots of oranges left, one lot containing 85,619 oranges, and the other 89,756. How many oranges had he at first?

12. Divide:

a. $\frac{25}{27}$ by 5. *c.* $\frac{75}{98}$ by 15. *e.* $\frac{88}{103}$ by 11.

b. $\frac{7}{9}$ by 12. *d.* $\frac{11}{12}$ by 6. *f.* $\frac{450}{533}$ by 75.

13. Divide:

a. $\frac{9}{16}$ by 6. *c.* $\frac{24}{47}$ by 16. *e.* $\frac{1320}{2277}$ by 9.
b. $\frac{1728}{1845}$ by 36. *d.* $\frac{18}{35}$ by 12. *f.* $\frac{144}{155}$ by 60.

14. Divide:

a. 7 by $\frac{4}{5}$. *b.* 14 by $\frac{7}{9}$. *c.* 36 by $\frac{8}{4}$. *d.* 81 by $\frac{9}{10}$.

15. A man bought a million bricks. He used 8609 in building a wall, 87,609 in building a house, and 20,760 in building a stable. How many did he have left?

16. A merchant paid \$12.50 for figs, \$17.35 for raisins, \$29$\frac{1}{8}$ for spice, \$164.95 for sugar, \$716.29 for tea, and \$690 for coffee. He had \$2700 before paying for his goods. How much money had he left?

17. A family's expenses for one month were: baker's bill, \$5.73; butcher's bill, \$10.91; groceries, \$12.75; fruit, \$3.29; rent, \$26.25; servant's wages, \$16; tailor's account, \$17.87; shoemaker's bill, \$11.63; coal, \$5.75; and sundries, \$19.47. What was the total account for the month?

18. Subtract:

a.	*b.*	*c.*	*d.*	*e.*	*f.*
9864	7426	6239	4729	6258	4625
3712	3818	4127	501	36	1846
g.	*h.*	*i.*	*j.*	*k.*	
13602	20607	30102	31607	40102	
11795	19089	18607	19089	19007	

19. Add the numbers in **18**.

20. Draw a circle with arcs of these degrees: 60°, 30°, 45°, 180°.

What fractional part of the circumference is each arc?

21. Find the prime factors in: 865, 939, 1246.

22. Draw a parallelogram, and measure it.

23. Draw a triangle, and measure it.

24. Estimate their areas by such scales as:

$\frac{1}{8}$ in. = 1 ft. $\frac{1}{16}$ in. = 1 yd. $\frac{1}{4}$ in. = 1 mi.

REVIEW

1. To the difference between 17 and 6 add 8.

2. From the difference between 17 and 6 take 5.

3. To 6 add the sum of 3, 4, and 5.

4. From the sum of 13 and 5 take the sum of 12 and 6.

5. From the sum of 13 and 5 take 7.

6. To the difference between 15 and 8 add the sum of 2 and 3.

7. To the sum of 6, 4, and 5 add 7.

8. To 3 add the difference between 17 and 12.

9. To the difference between 12 and 5 add 5.

10. Add:

a. $8 + 3 + 1$.	*e*. $9 + 2 + 4$.	*i*. $7 + 6 + 5$.
b. $12 + 5 + 10$.	*f*. $15 + 2 + 4$.	*j*. $13 + 7 + 8$.
c. $65 + 40$.	*g*. $54 + 38$.	*k*. $29 + 61$.
d. $10 + 8 + 3$.	*h*. $5 + 8 + 9$.	*l*. $9 + 8 + 5$.

11. A man having \$56 bought a hat for \$3, a coat for \$10, and a pair of shoes for \$4. How much had he left?

12. How many are $87 - 2 - 3 - 4 - 5 - [9 - 8 + 7]$?

13. If $\frac{1}{8}$ A. of land cost \$10, what will $\frac{7}{8}$ A. cost?

14. If $\frac{1}{3}$ of a car load of peaches cost \$200, what will a whole car load cost?

15. What is the ratio of 10 bu. to 4 pk? of 2 pk. to 5 bu.?

16. What is the ratio of a mile to 1320 ft.?

17. What is the area of a right angle triangle with base 6 ft. and altitude 5 ft.?

18. What are *parallel* lines?

19. What is an *acute* angle?

20. $2 : 3 :: 8 : x$. What is x?

A.	B.	C.	D.	E.
87406	92674	25043	179122	262083
41299	27049	97069	7626	198568
47208	28372	81216	6293	4318
71615	37041	90414	7404	39665
72428	49741	19216	30359	26881
28907	57214	72859	16660	34705
27426	92742	87604	80901	95299
87489	17618	27042	17255	80597
97206	21859	72687	32543	53806
25412	51814	18972	56063	8624
70412	59261	59876	40022	35077
62084	40216	32018	17548	29997
21642	42673	54301	16147	39744
81697	57274	71215	38556	44305
72204	87614	21441	234882	8735
41278	41219	60594	152526	169220

Proofs. Beginning with the unit's column, add each column, up and down. Or, separate the numbers into two or more sets, find the sum of each set, and add the sums.

107485075	
870302861	
73628874	
668279	
47538234	
267849026	
486792836	
9563748	
6473928	
1870302861	total sum.
1051416810	
316055539	
502830512	

107485075	
870302861	
73628874	
1051416810	first partial sum.
668279	
47538234	
267849026	
316055539	second partial sum.
486792836	
9563748	
6473928	
502830512	third partial sum.

REVIEW

1. Add each combination. Subtract. Multiply.

13	15	14	13	12	11	12	16	11
5	7	7	7	5	5	5	7	4
15	14	10	16	12	12	14	12	17
7	5	3	8	9	8	5	2	8
13	11	10	12	17	13	11	17	10
5	4	8	7	9	8	9	7	6
13	11	15	14	13	11	13	14	10
8	3	8	7	5	2	8	6	7
13	12	15	11	13	10	14	12	16
6	5	8	2	7	9	5	3	8
13	11	12	14	10	13	17	13	15
8	6	7	9	2	6	9	8	1
10	13	14	17	11	12	16	12	14
6	9	7	8	9	8	7	5	9
14	16	11	10	13	17	11	10	18
8	9	5	6	4	9	7	4	9
10	14	15	12	18	15	14	10	13
6	9	7	6	9	6	8	5	4

2. In these combinations divide each larger number by the smaller. What fraction do we get in each case by calling the smaller number the numerator, the larger the divisor?

3. If meat is $12\frac{1}{2}$ cents per lb., what will 6 lb. cost? 10 lb.?

4. If 1 yd. of braid is worth 12¢, what are $4\frac{1}{2}$ yd. worth?

5. When rice is sold for 9¢ a pound, what must be paid for $7\frac{1}{2}$ pounds? for $8\frac{1}{2}$ pounds? for $9\frac{1}{2}$ pounds?

6. In the following examples each cost price is to be multiplied by each rate of profit or loss:

Cost Price.	Rate of Profit or Loss.
I $100.	*a.* 5% loss.
II $500.	*b.* 10% profit.
III $20.	*c.* 20% loss.
IV $350.	*d.* 30% profit.
V $120.	*e.* 5% profit.

Find the total gain or loss.

7. If a boy earns $.875 in 1 day, how much will he earn in $9\frac{1}{4}$ days?

8. If one A. of land cost $48, what will $\frac{5}{8}$ A. cost?

9. I owe one man $245, and another man $325, a third man $187, and a fourth man $96. How much do I owe the four men?

10. If a yard of cloth cost $1\frac{5}{8}$, what will $9\frac{1}{4}$ yd. cost? $\frac{3}{4}$ yd.? 3 yd.? $12\frac{1}{2}$ yd.?

11. Multiply:

a. .23 by .009. *b.* 26.087 by .408. *c.* 341.45 by .007.
d. 7.02 by 5.27. *e.* .3045 by .00061. *f.* .3020 by .015.
g. .004 by .04. *h.* 6.23 by .049. *i.* .132 by .241.

12. If $\frac{5}{9}$ of a bbl. of apples cost $1\frac{1}{2}$, what will a bbl. cost? 2 bbl.? 10 bbl.?

13. The length of the Danube is about 1750 mi.; of the Dnieper, 1250 mi.; of the Don, 1120 mi.; of the Rhine, 700 mi.; of the Elbe, 800 mi.; and of the Rhone, 650 mi. What is their aggregate length?

14. The length of the Amazon River is 3600 mi.; of the Missouri-Mississippi, 5500 mi.; of the Rio de la Plata, 2240 mi.; of the MacKenzie, 2000 mi.; of the St. Lawrence, 2100 mi.; of the Orinoco, 1600 mi.; and of the Rio Grande, 2150 mi. What is their aggregate length?

REVIEW

1. A grocer spent $\$56\frac{2}{5}$ for canned fruits, giving $\$1\frac{1}{5}$ a dozen cans. How many dozen cans did he buy?

2. Multiply sixty-five thousandths by two hundred and twenty thousandths.

3. A roller 4.12 ft. in circumference makes 35.2 revolutions in passing from one end of a lawn to another. What is the length of the lawn?

4. From 1 A. of land I sold two house lots, each 100 sq. ft. What is the value of the rest of the land at $8\cent$ per sq. ft.?

5. How much does $.06896 \times 1.45$ differ from 1?

$.00674 \times 321$? $.095 \times 1000$? 25.238×12.17?

$.23 \times .25$? $.00514 \times 867.59$? 4.285×17.05?

6. If 1 ton of coal cost $\$4.275$, what will 300 tons cost?

7. In what time will 12 horses consume a stack of hay that will last 21 horses $6\frac{2}{7}$ wk.?

8. A dealer sold 372 lb. of raisins at $15\cent$ per lb., 434 lb. of currants at $12\frac{1}{2}\cent$ per lb., and 16 lb. of coffee at $\$.33$ per lb. What was the amount of these sales?

9. How many gallons are there in a cubic foot? There are 231 cu. in. in 1 gal.

10. Answer:

a. $33000 \div 16$. *b*. $12300 \div 25$. *c*. $5439 \div 49$.

d. $12771 \div 27$. *e*. $25830 \div 35$. *f*. $34104 \div 56$.

g. $452160 \div 64$. *h*. $388575 \div 55$. *i*. $2953216 \div 128$.

11. Write ten numbers to be added together, using billions, trillions, and quadrillions.

12. Subtract two numbers, one in decillions, the other in octillions.

13. Write a very, very small fraction in decimals.

14. I lost 10 per cent of $975. How much had I left?

15. Find: *a.* $46109 \div 49$. *d.* $135791 \div 125$.
b. $100000 \div 165$. *e.* $479102 \div 165$.
c. $975319 \div 616$. *f.* $63844 \div 135$.

16. What will 79 bu. of wheat cost at $\$\frac{7}{8}$ per bu.?

17. Find the cost of $4\frac{3}{8}$ bu. of peaches at $1.75 per bu.?

18. Find the cost of 7 tons of hay at $\$23\frac{6}{8}$ per ton.

19. If a man receives $\frac{3}{8}$ of a dollar for one hour's labor, what will he receive for 21 hours' labor?

20. Reduce to common denominator and add:

a. $\frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{7}$.
b. $\frac{5}{6}, \frac{5}{8}, \frac{2}{9}, \frac{13}{14}$.
c. $\frac{11}{12}, \frac{9}{16}, \frac{13}{72}, \frac{2}{9}, \frac{13}{18}, \frac{7}{20}$.
d. $\frac{1}{5}, \frac{2}{3}, \frac{7}{10}, \frac{5}{6}$.
e. $\frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \frac{31}{32}$.
f. $\frac{1}{10}, \frac{5}{16}, \frac{7}{24}, \frac{8}{35}, \frac{9}{40}, \frac{11}{48}$.
g. $\frac{3}{8}, \frac{5}{24}, \frac{7}{10}, \frac{5}{32}, \frac{11}{48}$.
h. $\frac{9}{16}, \frac{8}{25}, \frac{33}{50}, \frac{23}{48}, \frac{11}{32}, \frac{1}{240}$.

21. One variety of Egyptian granite weighs 350 lb. per cu. ft. What is the weight of 1 cu. yd. of such granite?

22. I sent to Liverpool 5000 barrels of apples, which cost me $1.25 per barrel. 25 per cent of the apples were thrown overboard in a storm, and the remainder were sold at $2 per barrel. What was gained?

23. If 55 men earn a certain sum of money in 21 days, how many men can earn the same sum in 77 days?

24. How many days, at $1.50 a day, must 48 men work in order to pay for 240 bushels of potatoes at 60 ¢ a bushel?

25. A man worked for 20 days at $2.40 a day, and took in payment 60 bushels of wheat. How much was the wheat valued at a bushel?

26. $x : 5 = 30 : 20$. **27.** $2 : x = 24 : 9$.

28. What is the area of a rectangle 3 ft. by 10 yd.?

REVIEW

1. Give the Tables of Measures.

2. Give the aliquot parts of one hundred.

3. From 1 take .125. 4. From 63.25 take 50.

5. What are the factors of 18? of 32? of 144? of 150?

6. A farmer who had 80 sheep sold 20% of them. How many did he sell?

7. A man having $4000 invests 25 per cent of it. What sum does he invest?

8. What part of an area 3 yd. square is 3 sq. yd.?

9. If $\frac{5}{8}$ T. of coal cost $3.20, what will $\frac{15}{16}$ T. cost?

10. Find the cost of 5 tons of coal at $3\frac{1}{8}$ per ton.

11. If 2 dozen pears cost 50¢, what will 3 dozen cost?

12. How many bushels of corn at 60¢ a bushel can be exchanged for 12 bushels of potatoes at 55¢ a bushel?

13. I sold a horse for $300, gaining thereby $40.75. What did the horse cost me?

14. I bought a bicycle for $50, and sold it for 5% more than it cost me. How much did I gain?

15. If 6 men can build a boat in 120 da., how long will it take 24 men to build it?

16. If $\frac{1}{4}$ of $\frac{3}{5}$ of the cost of a city hall was $300,000, what was the whole cost?

17. If 7 men can pave a street in 21 da., how many men will be required to pave it in 3 da.?

18. A room is 12 ft. long, 12 ft. high, and 10 ft. wide. How many cubic feet are there in the room?

19. I bought grain for $425, and sold it at 100 per cent profit. What was my gain?

20. Draw on the blackboard a decagon, and divide it into ten equal triangles.

21. From 8960.320507 take 63.001.

22. From 75 hundredths take 75 thousandths.

23. Find the value of:

a. $37.5 + 48.26 + 0.00831 - 85.759$.

b. $2.02 - 0.0909 - 1.9009 + 19.009 - 9.029209$.

24. A piece of cloth measured $23\frac{5}{8}$ yd. before fulling, but only $21\frac{5}{6}$ yd. after it. How much did the cloth shrink?

25. A dealer bought 1500 bbl. of apples, and found 12% of the fruit was spoiled. How many bbl. did he lose?

26. Increase \$225 by 8% of itself.

27. Reduce to common fractions:

a. .75. *c.* .375. *e.* .4. *g.* .625. *i.* .21875.
b. .625. *d.* .415. *f.* .625 *h.* .008. *j.* .024.

28. What is 220% of 117 barrels of apples?

29. Reduce the fractions $\frac{1}{3}$, $\frac{2}{5}$, and $\frac{5}{7}$ to equivalent fractions having a common denominator.

30. Decrease \$360 by 6% of itself.

31. Two persons have \$24 between them; the first has three times as much as the second. How much has each?

32. Three boys are to share 24¢ among them; the second boy is to have twice as many as the first, and the third boy is to have three times as many as the first.

Let x represent the share of the first.

33. Wilson agreed to sell $37\frac{1}{2}$ cords of wood to Jackson. He delivered $9\frac{1}{4}$ cords one week, $8\frac{7}{8}$ the next, and $10\frac{5}{8}$ the next. How many cords then had he still to deliver?

34. Draw a trapezoid, giving its measures. Divide it into triangles, using dotted lines. Find its area.

35. What is the circumference of a circle with radius 2.3 in.? What is its diameter?

REVIEW

1. The greater of two numbers is equal to six times the smaller, and their sum is 49. Find the numbers.

2. Telegraph poles were placed 132 feet apart. How many feet were there between the first pole and the last of 236 such poles?

3. If a man wastes 6¢ a day, how much money will he waste in thirty years?

4. $\frac{13}{18} - \frac{8}{15} + \frac{11}{20} - \frac{13}{30} = ?$

5. Multiply 818,647,362 by 2, 3, 4, 5, 6, 7, 8, 9.

6. At $7\frac{1}{2}$¢ per yd., what will 43 yd. of braid cost?

7. If a gallon of kerosene burns $4\frac{5}{6}$ hours in a lamp, how long would 15 gallons burn?

8. I spent \$3.62½ for a vest; \$1¾ for a pocket-book; \$1⅜ for a knife; \$.62½ for a necktie; \$.87½ for a key ring. How much did I spend in all?

9. Two trains start at the same point and travel in opposite directions, one at the rate of 34 miles an hour, the other at the rate of 28 miles an hour. How far apart will the trains be at the end of 14 hours?

10. If a man borrows \$800 at 6% for 15 mo., at the end of the time how much money will he owe?

11. $\frac{15 \times 32 \times 84 + 3}{24 \times 9 \times 33 \times 60} = ?$

12. $\frac{3 + 6 \times 5 + 15 - 7}{7 - (8 - 5) \times 2 + 2} = ?$

13. $x = 5(6 - 3) + (8 - 4)(5 + 2)$. What is x?

14. $7 : 45 = 63 : x$.

15. $x : 9\% = \$100 : \$333.33$.

16. Draw a triangle with base $3\frac{1}{2}$ in. long and altitude 2 in. Find its area.

17. Draw a circle with a radius of 2 in.

REVIEW

1. What is 2 per cent of $750?

2. How many cubic feet are there in a rectangular bin 8 ft. by 5 ft. by 4 ft.?

3. If $\frac{7}{8}$ A. of land cost $21, what will $\frac{1}{8}$ A. cost?

4. What will 56 lbs. of salt cost at $\frac{3}{4}$¢ per lb.?

5. How many tons of coal at $6 a ton will pay for 30 cords of wood at $4 a cord?

6. Harry and Tom have 75¢, but Harry has twice as many cents as Tom. How many cents has each boy?

7. If M. feet of boards cost $12, what will 5 M. feet cost? 6 M.? 9 M.? 12 M.?

8. If 30 men can earn a certain sum of money in 25 days, how long will it take one man to earn it?

9. How many cubic feet are contained in a pile of cordwood 8 ft. 4 ft. 4 ft.?

10. What is the cost of $2\frac{1}{4}$ lb. of butter at 24¢ per lb.?

11. What are the factors of 48?

12. Find the greatest common divisor:

a. 8, 14, 12. *b.* 25, 10, 20. *c.* 15, 30, 35.
d. 3, 12, 6. *e.* 32, 8, 40. *f.* 18, 27, and 105.
g. 8, 12, 20. *h.* 60, 70, 80. *i.* 24, 12, 16.

13. Tell the meaning of each of these terms:

parallelogram	numerator	arc	angle
obtuse angle	denominator	decimal	interest
cancellation	perimeter	quotient	radius
proportion	equation	quadrant	factor
antecedent	consequent	perpendicular	exponent
greatest common divisor		symbol of aggregation	

REVIEW

1. What part of 63 is 1890? Answer in decimals.

2. $3\frac{3}{8} + 2\frac{5}{6} - \frac{7}{12} + 3\frac{4}{9} = ?$

3. A man bought 56 head of cattle for \$25 a head, and 94 for \$32 a head. If he sells the whole lot at \$30 a head, will he gain or lose, and how much?

4. A man whose yearly salary is \$3500 expends for rent \$600; for fuel and clothing, \$570; for provisions, \$635; and for other expenses, \$1200. How much money can he save in $11\frac{5}{12}$ years, the term of a series of stock in a local building and loan association?

5. If a man can build a wall in 32 days by working 8 hours a day, how many days would he take to build the same wall, working only 6 hours a day?

6. A dealer sold $17\frac{7}{12}$ tons of cannel coal for $\$187\frac{3}{8}$. At that rate what would $\frac{3}{4}$ of a ton cost?

7. What is the interest at 6% on \$450 for 90 days?

8. $8(3+4) - (10+5) + x = 45$. Find x.

9. Add: one hundred five ten-thousandths; one and thirty-seven hundredths; ten thousand eight hundred seventy-five millionths.

10. What kind of an angle measures 90°? Draw it.

11. Draw a parallelogram of any form or size and measure its area, using no fraction smaller than $\frac{1}{8}$.

12. Write a bill of goods with five purchases.

13. Write a problem, using facts connected with Linear Measure.

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